Chapter 13, Solution 1.

For coil 1, $L_1 - M_{12} + M_{13} = 6 - 4 + 2 = 4$

For coil 2, $L_2 - M_{21} - M_{23} = 8 - 4 - 5 = -1$

For coil 3, $L_3 + M_{31} - M_{32} = 10 + 2 - 5 = 7$

$$L_T = 4 - 1 + 7 = 10H$$

or $$L_T = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} + 2M_{12}$$

$L_T = 6 + 8 + 10 =$ **10H**

Chapter 13, Solution 2.

$L = L_1 + L_2 + L_3 + 2M_{12} - 2M_{23}\ 2M_{31}$

$= 10 + 12 + 8 + 2x6 - 2x6 - 2x4$

$=$ **22H**

Chapter 13, Solution 3.

$L_1 + L_2 + 2M = 250$ mH (1)

$L_1 + L_2 - 2M = 150$ mH (2)

Adding (1) and (2),

$$2L_1 + 2L_2 = 400 \text{ mH}$$

But, $L_1 = 3L_{2,}$, or $8L_2 + 400$, and $L_2 =$ **50 mH**

$$L_1 = 3L_2 = \textbf{150 mH}$$

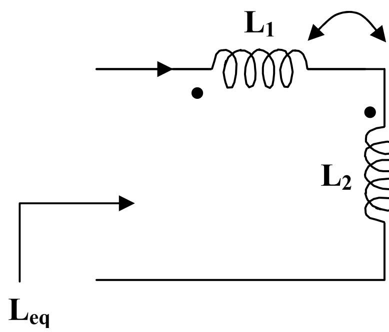
From (2), $150 + 50 - 2M = 150$ leads to $M =$ **25 mH**

$$k = M/\sqrt{L_1L_2} = 2.5/\sqrt{50x150} = \textbf{0.2887}$$

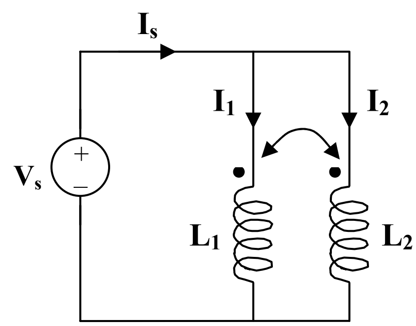
Chapter 13, Solution 4.

(a) For the series connection shown in Figure (a), the current I enters each coil from its dotted terminal. Therefore, the mutually induced voltages have the same sign as the self-induced voltages. Thus,

$$L_{eq} = L_1 + L_2 + 2M$$



(a)



(b)

(b) For the parallel coil, consider Figure (b).

$$I_s = I_1 + I_2 \quad \text{and} \quad Z_{eq} = V_s/I_s$$

Applying KVL to each branch gives,

$$V_s = j\omega L_1 I_1 + j\omega M I_2 \qquad (1)$$

$$V_s = j\omega M I_1 + j\omega\ L_2 I_2 \qquad (2)$$

or

$$\begin{bmatrix} V_s \\ V_s \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = -\omega^2 L_1 L_2 + \omega^2 M^2, \quad \Delta_1 = j\omega V_s(L_2 - M), \quad \Delta_2 = j\omega V_s(L_1 - M)$$

$$I_1 = \Delta_1/\Delta, \quad \text{and} \quad I_2 = \Delta_2/\Delta$$

$$I_s = I_1 + I_2 = (\Delta_1 + \Delta_2)/\Delta = j\omega(L_1 + L_2 - 2M)V_s/(\ -\omega^2(L_1L_2 - M))$$

$$Z_{eq} = V_s/I_s = j\omega(L_1L_2 - M)/[j\omega(L_1 + L_2 - 2M)] = j\omega L_{eq}$$

i.e., $L_{eq} = \underline{\mathbf{(L_1L_2 - M)/(L_1 + L_2 - 2M)}}$

Chapter 13, Solution 5.

(a) If the coils are connected in series,

$$L = L_1 + L_2 + 2M = 25 + 60 + 2(0.5)\sqrt{25x60} = \underline{\mathbf{123.7\ mH}}$$

(b) If they are connected in parallel,

$$L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{25x60 - 19.36^2}{25 + 60 - 2x19.36}\text{mH} = \underline{\mathbf{24.31\ mH}}$$

Chapter 13, Solution 6.

$V_1 = \underline{\mathbf{(R_1 + j\omega L_1)I_1 - j\omega MI_2}}$

$V_2 = \underline{\mathbf{-j\omega MI_1 + (R_2 + j\omega L_2)I_2}}$

Chapter 13, Solution 7.

Applying KVL to the loop,

$$20\angle 30^\circ = I(-j6 + j8 + j12 + 10 - j4x2) = I(10 + j6)$$

where I is the loop current.

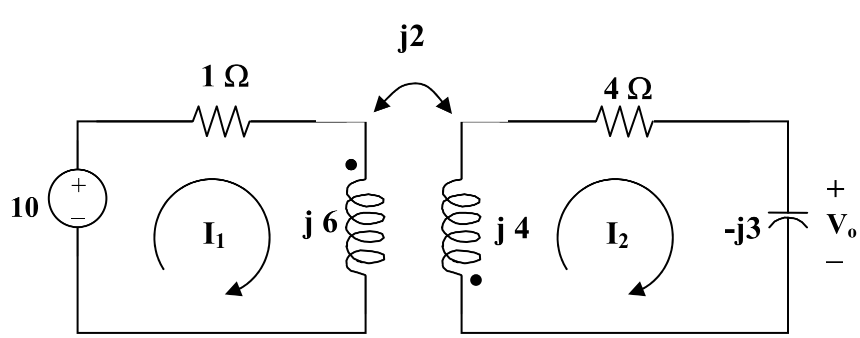
$$I = 20\angle 30°/(10 + j6)$$

$$V_o = I(j12 + 10 - j4) = I(10 + j8)$$

$$= 20\angle 30°(10 + j8)/(10 + j6) = \underline{\mathbf{22\angle 37.66°\ V}}$$

Chapter 13, Solution 8.

Consider the current as shown below.



For mesh 1,

$$10 = (1 + j6)I_1 + j2I_2 \qquad (1)$$

For mesh 2,

$$0 = (4 + j4 - j3)I_2 + j2I_1$$

$$0 = j2I_1 + (4 + j)I_2 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 10 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 + j6 & j2 \\ j2 & 4 + j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

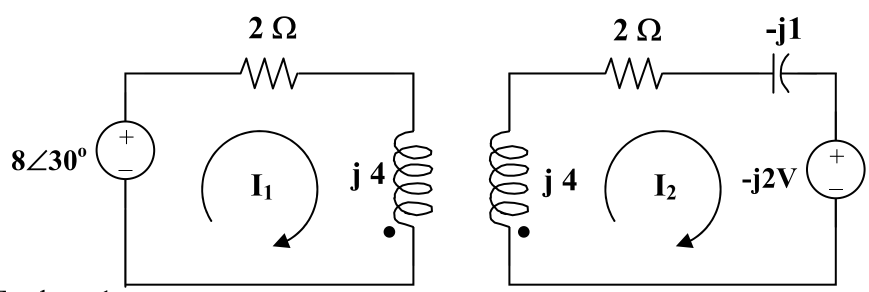
$$\Delta = 2 + j25, \text{ and } \Delta_2 = -j20$$

$$I_2 = \Delta_2/\Delta = -j20/(2 + j25)$$

$$V_o = -j3I_2 = -60/(2 + j25) = \underline{\mathbf{2.392\angle 94.57^\circ}}$$

Chapter 13, Solution 9.

Consider the circuit below.



For loop 1,

$$8\angle 30^\circ = (2 + j4)I_1 - jI_2 \qquad (1)$$

For loop 2,

$$((j4 + 2 - j)I_2 - jI_1 + (-j2) = 0$$

or

$$I_1 = (3 - j2)i_2 - 2 \qquad (2)$$

Substituting (2) into (1),

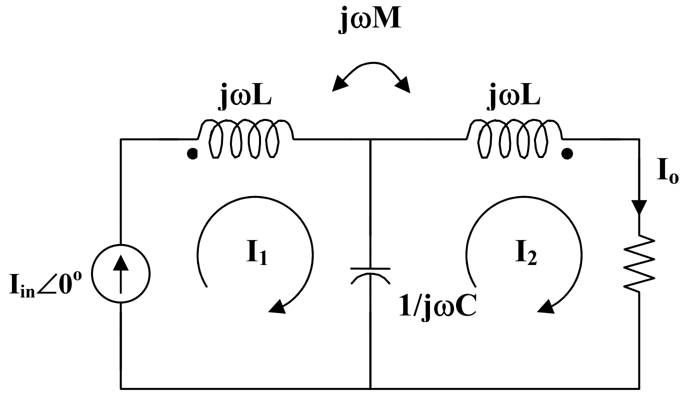
$$8\angle 30^\circ + (2 + j4)2 = (14 + j7)I_2$$

$$I_2 = (10.928 + j12)/(14 + j7) = 1.037\angle 21.12^\circ$$

$$V_x = 2I_2 = \underline{\mathbf{2.074\angle 21.12^\circ}}$$

Chapter 13, Solution 10.

Consider the circuit below.



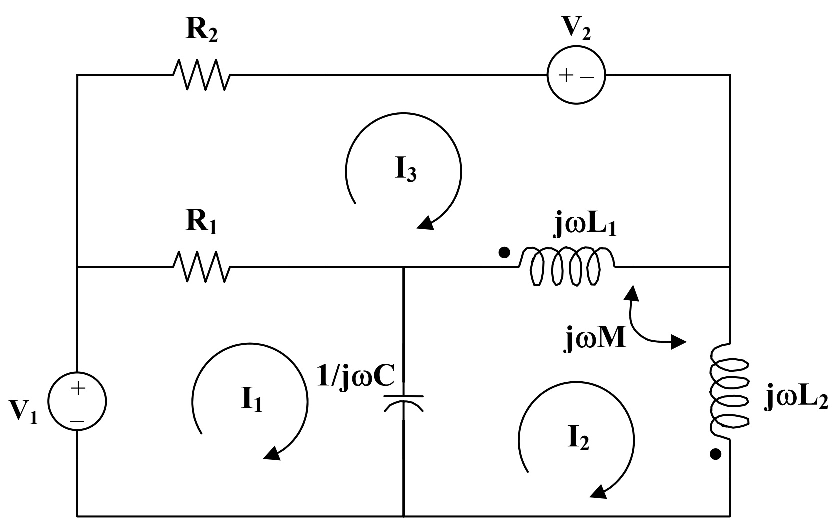
$M = k\sqrt{L_1 L_2} = \sqrt{L^2} = L,\ \ I_1 = I_{in}\angle 0°,\ \ I_2 = I_o$

$I_o(j\omega L + R + 1/(j\omega C)) - j\omega L I_{in} - (1/(j\omega C))I_{in} = 0$

$I_o = \underline{\mathbf{j\, I_{in}(\omega L - 1/(\omega C))\ /(R + j\omega L + 1/(j\omega C))}}$

Chapter 13, Solution 11.

Consider the circuit below.



For mesh 1, $V_1 = \underline{\mathbf{I_1(R_1 + 1/(j\omega C)) - I_2(1/j\omega C)) - R_1 I_3}}$

For mesh 2,

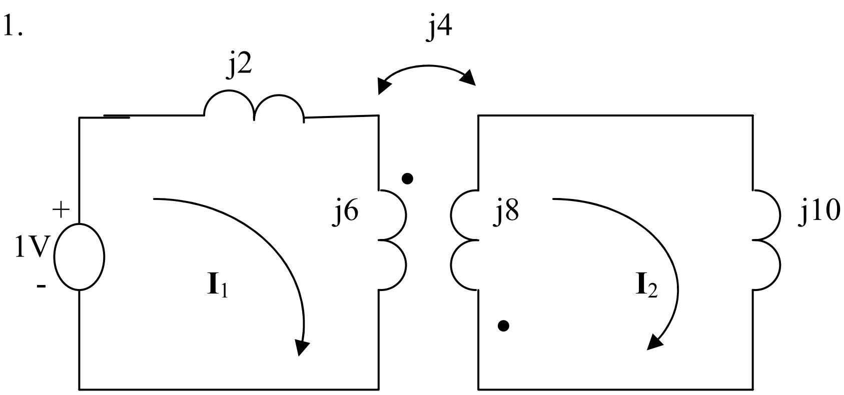
$0 = \underline{\mathbf{-I_1(1/(j\omega C)) + (j\omega L_1 + j\omega L_2 + (1/(j\omega C)) - j2\omega M)I_2 - j\omega L_1 I_3 + j\omega M I_3}}$

For mesh 3, $-V_2 = -R_1 I_1 - j\omega(L_1 - M)I_2 + (R_1 + R_2 + j\omega L_1)I_3$

or $V_2 = \underline{\mathbf{R_1 I_1 + j\omega(L_1 - M)I_2 - (R_1 + R_2 + j\omega L_1)I_3}}$

Chapter 13, Solution 12.

Let $\omega = 1$.



Applying KVL to the loops,

$1 = j8I_1 + j4I_2$ (1)

$0 = j4I_1 + j18I_2$ (2)

Solving (1) and (2) gives $\mathbf{I}_1$ = -j0.1406. Thus

$$Z = \frac{1}{I_1} = jL_{eq} \quad \longrightarrow \quad L_{eq} = \frac{1}{jI_1} = \underline{7.111 \text{ H}}$$

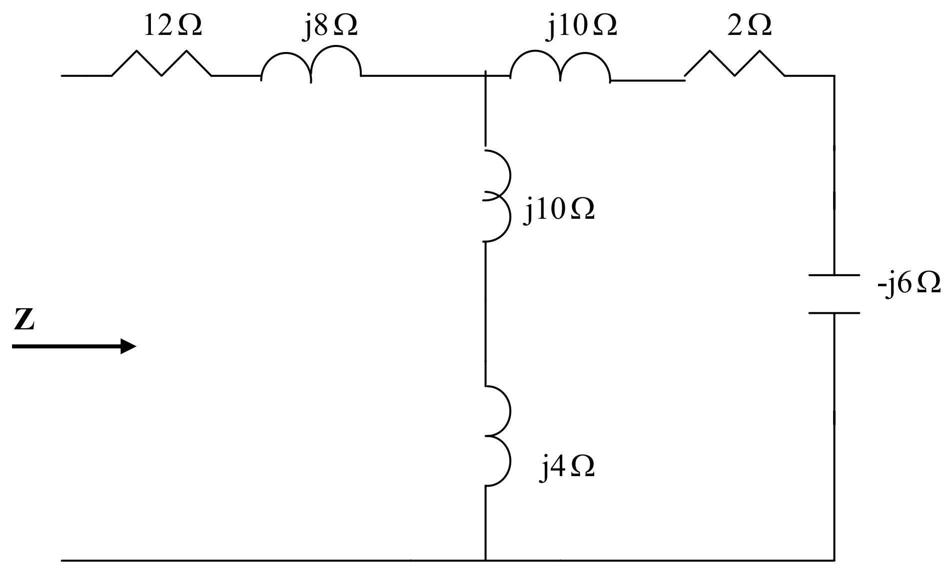
We can also use the equivalent T-section for the transform to find the equivalent inductance.

Chapter 13, Solution 13.

We replace the coupled inductance with an equivalent T-section and use series and parallel combinations to calculate **Z.** Assuming that $\omega = 1$,

$L_a = L_1 - M = 18 - 10 = 8, \qquad L_b = L_2 - M = 20 - 10 = 10, \qquad L_c = M = 10$

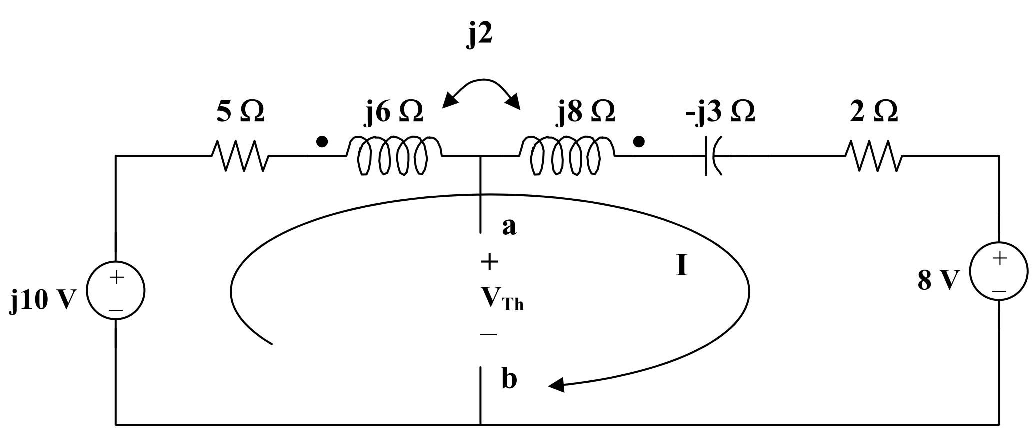
The equivalent circuit is shown below:



Z=12 +j8 + j14//(2 + j4) = **<u>13.195 + j11.244Ω</u>**

Chapter 13, Solution 14.

To obtain V_{Th}, convert the current source to a voltage source as shown below.



Note that the two coils are connected series aiding.

$$\omega L = \omega L_1 + \omega L_2 - 2\omega M$$

$$j\omega L = j6 + j8 - j4 = j10$$

Thus, $$-j10 + (5 + j10 - j3 + 2)I + 8 = 0$$

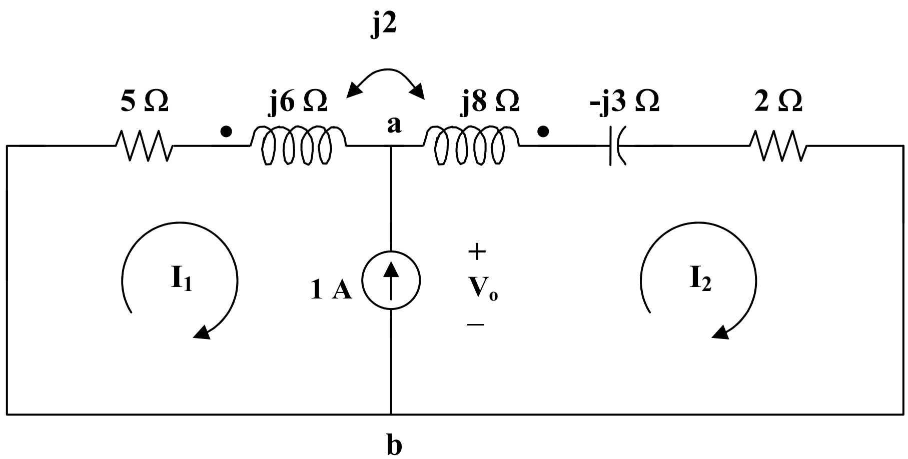
$$I = (-8 + j10)/ (7 + j7)$$

But, $$-j10 + (5 + j6)I - j2I + V_{Th} = 0$$

$$V_{Th} = j10 - (5 + j4)I = j10 - (5 + j4)(-8 + j10)/(7 + j7)$$

$$V_{Th} = \underline{\mathbf{5.349\angle 34.11^\circ}}$$

To obtain Z_{Th}, we set all the sources to zero and insert a 1-A current source at the terminals a–b as shown below.



Clearly, we now have only a super mesh to analyze.

$$(5 + j6)I_1 - j2I_2 + (2 + j8 - j3)I_2 - j2I_1 = 0$$

$$(5 + j4)I_1 + (2 + j3)I_2 = 0 \qquad (1)$$

But, $$I_2 - I_1 = 1 \text{ or } I_2 = I_1 - 1 \qquad (2)$$

Substituting (2) into (1), $$(5 + j4)I_1 + (2 + j3)(1 + I_1) = 0$$

$$I_1 = -(2 + j3)/(7 + j7)$$

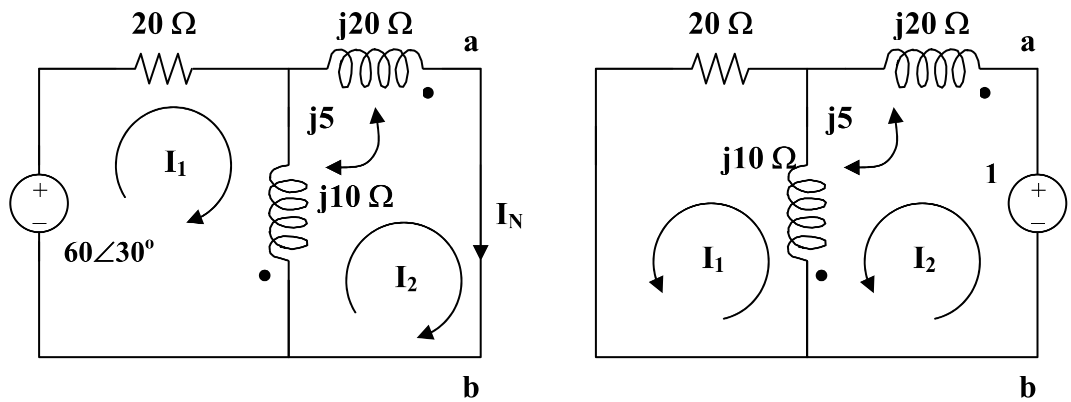
Now, $$((5 + j6)I_1 - j2I_1 + V_o = 0$$

$$V_o = -(5 + j4)I_1 = (5 + j4)(2 + j3)/(7 + j7) = (-2 + j23)/(7 + j7) = 2.332\angle 50^\circ$$

$$Z_{Th} = V_o/1 = \underline{\mathbf{2.332\angle 50^\circ \text{ ohms}}}$$

Chapter 13, Solution 15.

To obtain I_N, short-circuit a–b as shown in Figure (a).



(a) **(b)**

For mesh 1,

$$60\angle 30° = (20 + j10)I_1 + j5I_2 - j10I_2$$

or $$12\angle 30° = (4 + j2)I_1 - jI_2 \quad (1)$$

For mesh 2,

$$0 = (j20 + j10)I_2 - j5I_1 - j10I_1$$

or $$I_1 = 2I_2 \quad (2)$$

Substituting (2) into (1), $$12\angle 30° = (8 + j3)I_2$$

$$I_N = I_2 = 12\angle 30°/(8 + j3) = \underline{\mathbf{1.404\angle 9.44°\ A}}$$

To find Z_N, we set all the sources to zero and insert a 1-volt voltage source at terminals a–b as shown in Figure (b).

For mesh 1, $$1 = I_1(j10 + j20 - j5x2) + j5I_2$$

$$1 = j20I_1 + j5I_2 \quad (3)$$

For mesh 2, $$0 = (20 + j10)I_2 + j5I_1 - j10I_1 = (4 + j2)I_2 - jI_1$$

or $$I_2 = jI_1/(4 + j2) \quad (4)$$

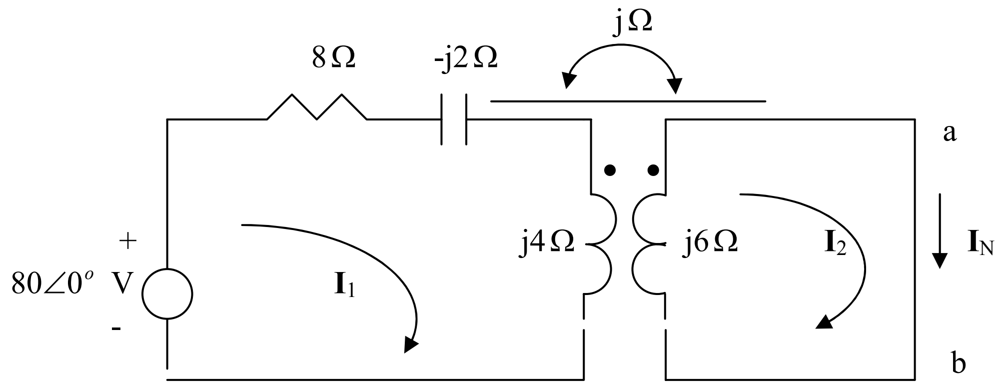
Substituting (4) into (3), $$1 = j20I_1 + j(j5)I_1/(4 + j2) = (-1 + j20.5)I_1$$

$$I_1 = 1/(-1 + j20.5)$$

$$Z_N = 1/I_1 = \underline{\mathbf{(-1 + j20.5)\ ohms}}$$

Chapter 13, Solution 16.

To find $\mathbf{I}_N$, we short-circuit a-b.

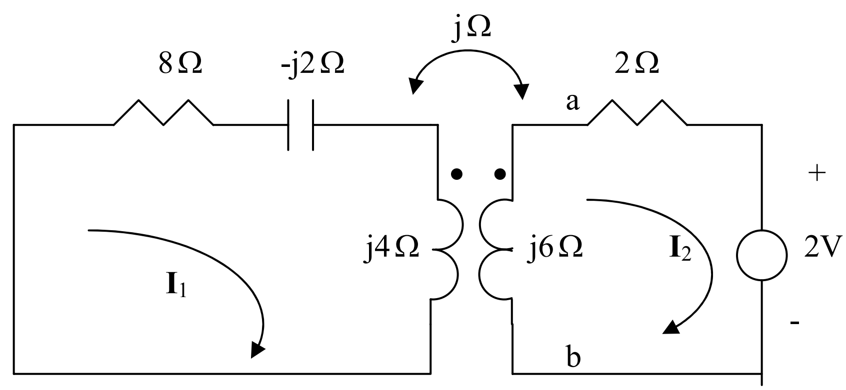


$$-80+(8-j2+j4)I_1 - jI_2 = 0 \longrightarrow (8+j2)I_1 - jI_2 = 80 \quad (1)$$
$$j6I_2 - jI_1 = 0 \longrightarrow I_1 = 6I_2 \quad (2)$$

Solving (1) and (2) leads to

$$I_N = I_2 = \frac{80}{48+j11} = 1.584 - j0.362 = \underline{1.6246\angle -12.91^o \text{ A}}$$

To find Z_N, insert a 1-A current source at terminals a-b. Transforming the current source to voltage source gives the circuit below.



$$0 = (8+j2)I_1 - jI_2 \longrightarrow I_1 = \frac{jI_2}{8+j2} \quad (3)$$

$$2+(2+j6)I_2 - jI_1 = 0 \quad (4)$$

Solving (3) and (4) leads to $\mathbf{I}_2$ = -0.1055 +j0.2975, $\mathbf{V}_{ab}$=-j6$\mathbf{I}_2$ = 1.7853 +0.6332

$$Z_N = \frac{V_{ab}}{1} = \underline{1.894\angle 19.53^o\,\Omega}$$

Chapter 13, Solution 17.

$$Z = -j6 \,//\, Z_o$$

where

$$Z_o = j20 + \frac{144}{j30 - j2 + j5 + 4} = 0.5213 + j15.7$$

$$Z = \frac{-j6xZ_o}{-j6 + Z_o} = \underline{0.1989 - j9.7\Omega}$$

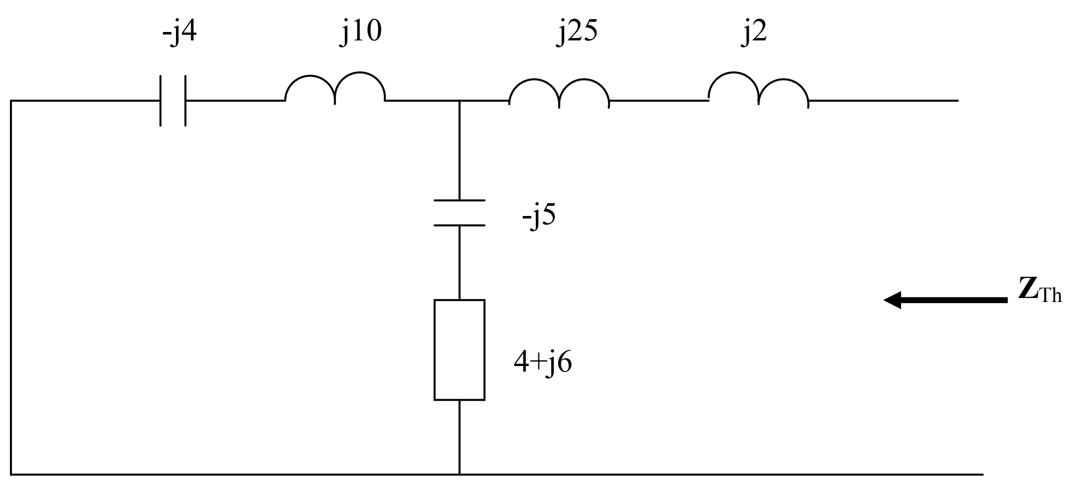
Chapter 13, Solution 18.

Let $\omega = 1.$ $L_1 = 5, L_2 = 20, M = k\sqrt{L_1 L_2} = 0.5x10 = 5$

We replace the transformer by its equivalent T-section.

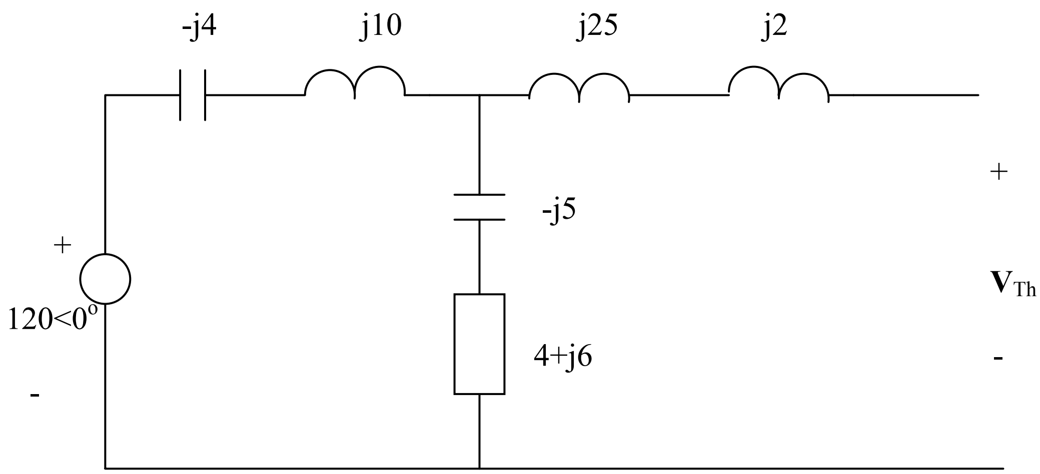
$L_a = L_1 - (-M) = 5 + 5 = 10,$ $L_b = L_1 + M = 20 + 5 = 25,$ $L_c = -M = -5$

We find $\mathbf{Z}_{Th}$ using the circuit below.



$$Z_{Th} = j27 + (4 + j)//(j6) = j27 + \frac{j6(4 + j)}{4 + j7} = \underline{2.215 + j29.12\Omega}$$

We find $\mathbf{V}_{Th}$ by looking at the circuit below.



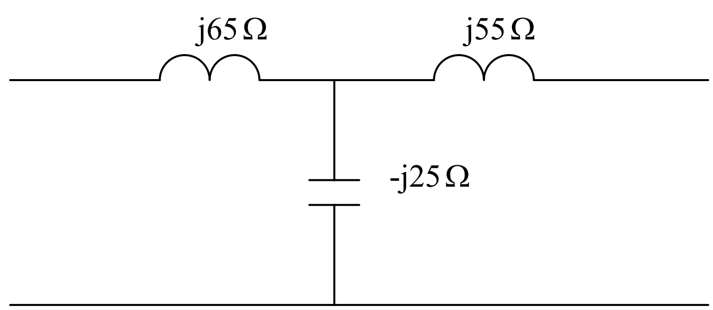
$$V_{Th} = \frac{4+j}{4+j+j6}(120) = \underline{61.37\angle -46.22^o \text{ V}}$$

Chapter 13, Solution 19.

Let $\omega = 1$. $L_a = L_1 - (-M) = 40 + 25 = 65$ H

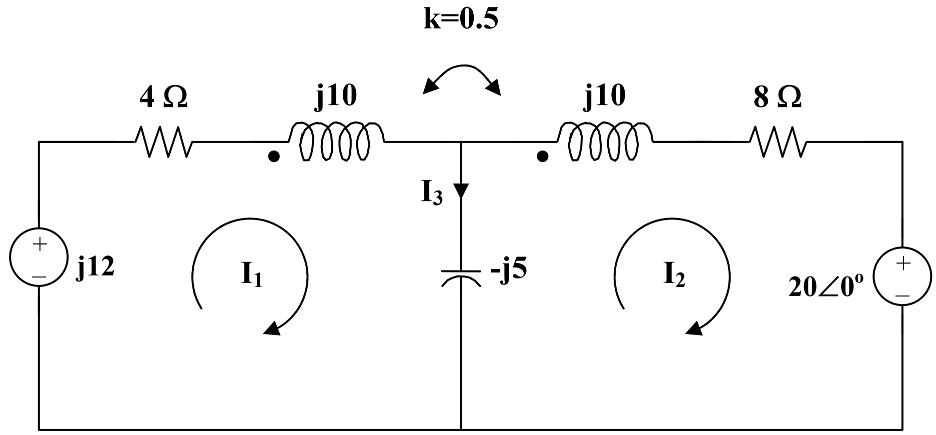
$L_b = L_2 + M = 30 + 25 = 55$ H, $L_C = -M = -25$

Thus, the T-section is as shown below.



Chapter 13, Solution 20.

Transform the current source to a voltage source as shown below.



$$k = M/\sqrt{L_1L_2} \quad \text{or} \quad M = k\sqrt{L_1L_2}$$

$$\omega M = k\sqrt{\omega L_1 \omega L_2} = 0.5(10) = 5$$

For mesh 1, $\quad j12 = (4 + j10 - j5)I_1 + j5I_2 + j5I_2 = (4 + j5)I_1 + j10I_2 \qquad (1)$

For mesh 2, $\quad 0 = 20 + (8 + j10 - j5)I_2 + j5I_1 + j5I_1$

$$-20 = +j10I_1 + (8 + j5)I_2 \qquad (2)$$

From (1) and (2), $\begin{bmatrix} j12 \\ 20 \end{bmatrix} = \begin{bmatrix} 4 + j5 & +j10 \\ +j10 & 8 + j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\Delta = 107 + j60, \quad \Delta_1 = -60 - j296, \quad \Delta_2 = 40 - j100$$

$$I_1 = \Delta_1/\Delta = \underline{\mathbf{2.462\angle 72.18^\circ\ A}}$$

$$I_2 = \Delta_2/\Delta = \underline{\mathbf{0.878\angle -97.48^\circ\ A}}$$

$$I_3 = I_1 - I_2 = \underline{\mathbf{3.329\angle 74.89^\circ\ A}}$$

$$i_1 = 2.462 \cos(1000t + 72.18^\circ)\ A$$

$$i_2 = 0.878 \cos(1000t - 97.48^\circ)\ A$$

At $t = 2$ ms, $1000t = 2$ rad $= 114.6°$

$$i_1 = 0.9736\cos(114.6° + 143.09°) = -2.445$$

$$i_2 = 2.53\cos(114.6° + 153.61°) = -0.8391$$

The total energy stored in the coupled coils is

$$w = 0.5L_1 i_1^2 + 0.5L_2 i_2^2 - M i_1 i_2$$

Since $\omega L_1 = 10$ and $\omega = 1000$, $L_1 = L_2 = 10$ mH, $M = 0.5L_1 = 5$mH

$$w = 0.5(10)(-2.445)^2 + 0.5(10)(-0.8391)^2 - 5(-2.445)(-0.8391)$$

w = 43.67 mJ

Chapter 13, Solution 21.

For mesh 1, $36\angle 30° = (7 + j6)I_1 - (2 + j)I_2$ (1)

For mesh 2, $0 = (6 + j3 - j4)I_2 - 2I_1 \; jI_1 = -(2 + j)I_1 + (6 - j)I_2$ (2)

Placing (1) and (2) into matrix form, $\begin{bmatrix} 36\angle 30° \\ 0 \end{bmatrix} = \begin{bmatrix} 7 + j6 & -2 - j \\ -2 - j & 6 - j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

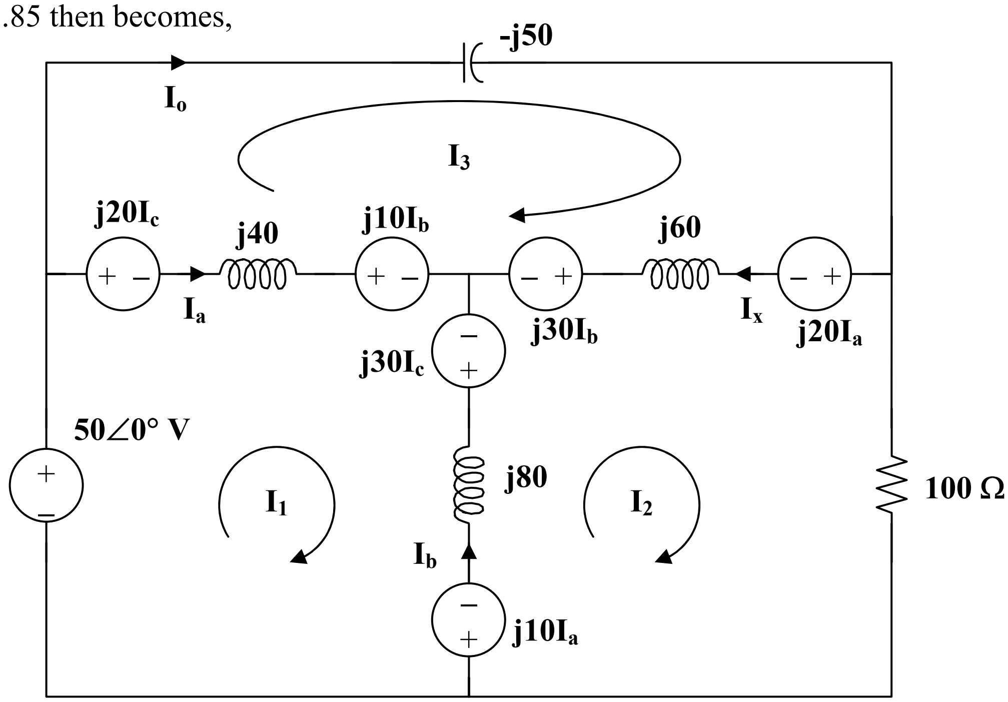
$\Delta = 48 + j35 = 59.41\angle 36.1°$, $\Delta_1 = (6 - j)36\angle 30° = 219\angle 20.54°$

$\Delta_2 = (2 + j)36\angle 30° = 80.5\angle 56.56°$, $I_1 = \Delta_1/\Delta = 3.69\angle -15.56°$, $I_2 = \Delta_2/\Delta = 1.355\angle 20.46°$

Power absorbed fy the 4-ohm resistor, $= 0.5(I_2)^2 4 = 2(1.355)^2 =$ **3.672 watts**

Chapter 13, Solution 22.

With more complex mutually coupled circuits, it may be easier to show the effects of the coupling as sources in terms of currents that enter or leave the dot side of the coil. Figure 13.85 then becomes,



Note the following,

$$I_a = I_1 - I_3$$
$$I_b = I_2 - I_1$$
$$I_c = I_3 - I_2$$

$$\text{and} \quad I_o = I_3$$

Now all we need to do is to write the mesh equations and to solve for I_o.

Loop # 1,

$$-50 + j20(I_3 - I_2)\ j\ 40(I_1 - I_3) + j10(I_2 - I_1) - j30(I_3 - I_2) + j80(I_1 - I_2) - j10(I_1 - I_3) = 0$$

$$j100I_1 - j60I_2 - j40I_3 = 50$$

Multiplying everything by (1/j10) yields $10I_1 - 6I_2 - 4I_3 = -j5$ (1)

Loop # 2,

$$j10(I_1 - I_3) + j80(I_2 - I_1) + j30(I_3 - I_2) - j30(I_2 - I_1) + j60(I_2 - I_3) - j20(I_1 - I_3) + 100I_2 = 0$$

$$-j60I_1 + (100 + j80)I_2 - j20I_3 = 0 \qquad (2)$$

Loop # 3,

$$-j50I_3 + j20(I_1 - I_3) + j60(I_3 - I_2) + j30(I_2 - I_1) - j10(I_2 - I_1) + j40(I_3 - I_1) - j20(I_3 - I_2) = 0$$

$$-j40I_1 - j20I_2 + j10I_3 = 0$$

Multiplying by (1/j10) yields, $-4I_1 - 2I_2 + I_3 = 0$ (3)

Multiplying (2) by (1/j20) yields $-3I_1 + (4 - j5)I_2 - I_3 = 0$ (4)

Multiplying (3) by (1/4) yields $-I_1 - 0.5I_2 - 0.25I_3 = 0$ (5)

Multiplying (4) by (-1/3) yields $I_1 - ((4/3) - j(5/3))I_2 + (1/3)I_3 = -j0.5$ (7)

Multiplying [(6)+(5)] by 12 yields $(-22 + j20)I_2 + 7I_3 = 0$ (8)

Multiplying [(5)+(7)] by 20 yields $-22I_2 - 3I_3 = -j10$ (9)

(8) leads to $I_2 = -7I_3/(-22 + j20) = 0.2355\angle 42.3^\circ = (0.17418 + j0.15849)I_3$ (10)

(9) leads to $I_3 = (j10 - 22I_2)/3$, substituting (1) into this equation produces,

$$I_3 = j3.333 + (-1.2273 - j1.1623)I_3$$

or $I_3 = I_o =$ **<u>1.3040∠63° amp.</u>**

Chapter 13, Solution 23.

$\omega = 10$

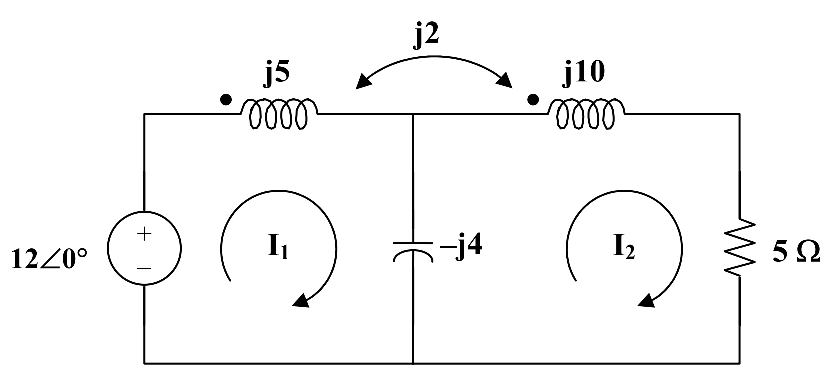
0.5 H converts to $j\omega L_1 = j5$ ohms

1 H converts to $j\omega L_2 = j10$ ohms

0.2 H converts to $j\omega M = j2$ ohms

25 mF converts to $1/(j\omega C) = 1/(10 \times 25 \times 10^{-3}) = -j4$ ohms

The frequency-domain equivalent circuit is shown below.



For mesh 1, $12 = (j5 - j4)I_1 + j2I_2 - (-j4)I_2$

$$-j2 = I_1 + 6I_2 \qquad (1)$$

For mesh 2, $0 = (5 + j10)I_2 + j2I_1 - (-j4)I_1$

$$0 = (5 + j10)I_2 + j6I_1 \qquad (2)$$

From (1), $I_1 = -j12 - 6I_2$

Substituting this into (2) produces,

$$I_2 = 72/(-5 + j26) = 2.7194\angle -100.89°$$

$$I_1 = -j12 - 6\,I_2 = -j12 - 163.17\angle -100.89 = 5.068\angle 52.54°$$

Hence, i_1 = **<u>5.068cos(10t + 52.54°) A</u>**, i_2 = **<u>2.719cos(10t – 100.89°) A</u>**.

At t = 15 ms, $10t = 10x15x10^{-3}\ 0.15 \text{ rad} = 8.59°$

$$i_1 = 5.068\cos(61.13°) = 2.446$$

$$i_2 = 2.719\cos(-92.3°) = -0.1089$$

$$w = 0.5(5)(2.446)^2 + 0.5(1)(-0.1089)^2 - (0.2)(2.446)(-0.1089) = \underline{\mathbf{15.02\ J}}$$

Chapter 13, Solution 24.

(a) $k = M/\sqrt{L_1L_2} = 1/\sqrt{4x2} = \underline{\mathbf{0.3535}}$

(b) $\omega = 4$

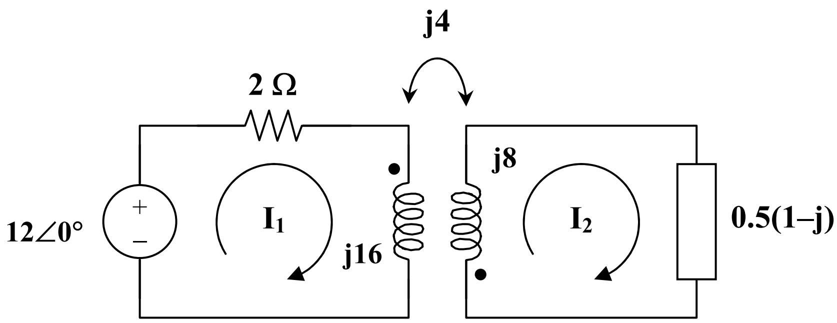
1/4 F leads to $1/(j\omega C) = -j/(4x0.25) = -j$

$1||(-j) = -j/(1 - j) = 0.5(1 - j)$

1 H produces $j\omega M = j4$

4 H produces j16

2 H becomes j8



$$12 = (2 + j16)I_1 + j4I_2$$

$$\text{or} \qquad 6 = (1 + j8)I_1 + j2I_2 \qquad (1)$$

$$0 = (j8 + 0.5 - j0.5)I_2 + j4I_1 \text{ or } I_1 = (0.5 + j7.5)I_2/(-j4) \qquad (2)$$

Substituting (2) into (1),

$$24 = (-11.5 - j51.5)I_2 \text{ or } I_2 = -24/(11.5 + j51.5) = -0.455\angle -77.41°$$

$$V_o = I_2(0.5)(1 - j) = 0.3217\angle 57.59°$$

$$v_o = \underline{\mathbf{321.7cos(4t + 57.6°) mV}}$$

(c) From (2), $I_1 = (0.5 + j7.5)I_2/(-j4) = 0.855\angle -81.21°$

$$i_1 = 0.885\cos(4t - 81.21°) \text{ A}, \ i_2 = -0.455\cos(4t - 77.41°) \text{ A}$$

At t = 2s,

$$4t = 8 \text{ rad} = 98.37°$$

$$i_1 = 0.885\cos(98.37° - 81.21°) = 0.8169$$

$$i_2 = -0.455\cos(98.37° - 77.41°) = -0.4249$$

$$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 + Mi_1i_2$$

$$= 0.5(4)(0.8169)^2 + 0.5(2)(-.4249)^2 + (1)(0.1869)(-0.4249) = \underline{\mathbf{1.168 J}}$$

Chapter 13, Solution 25.

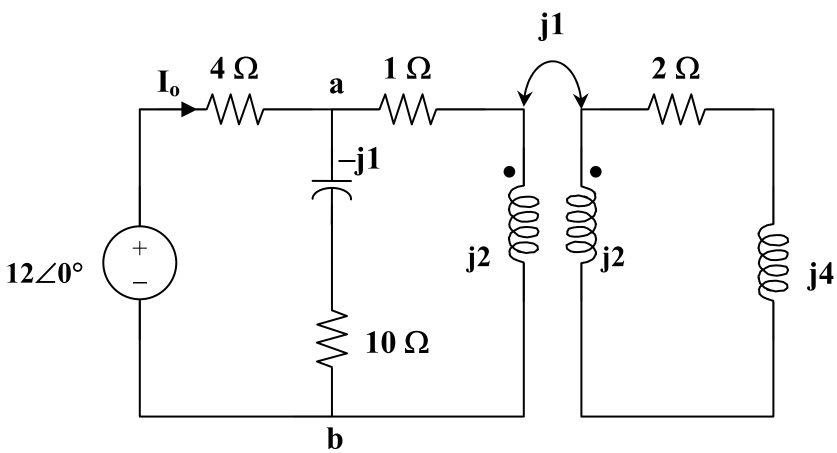
$$m = k\sqrt{L_1L_2} = 0.5 \text{ H}$$

We transform the circuit to frequency domain as shown below.

12sin2t converts to $12\angle 0°$, $\omega = 2$

0.5 F converts to $1/(j\omega C) = -j$

2 H becomes $j\omega L = j4$



Applying the concept of reflected impedance,

$$Z_{ab} = (2-j)||(1 + j2 + (1)^2/(j2 + 3 + j4))$$

$$= (2-j)||(1 + j2 + (3/45) - j6/45)$$

$$= (2-j)||(1 + j2 + (3/45) - j6/45)$$

$$= (2-j)||(1.0667 + j1.8667)$$

$$=(2-j)(1.0667 + j1.8667)/(3.0667 + j0.8667) = 1.5085\angle 17.9° \text{ ohms}$$

$$I_o = 12\angle 0°/(Z_{ab} + 4) = 12/(5.4355 + j0.4636) = 2.2\angle{-4.88°}$$

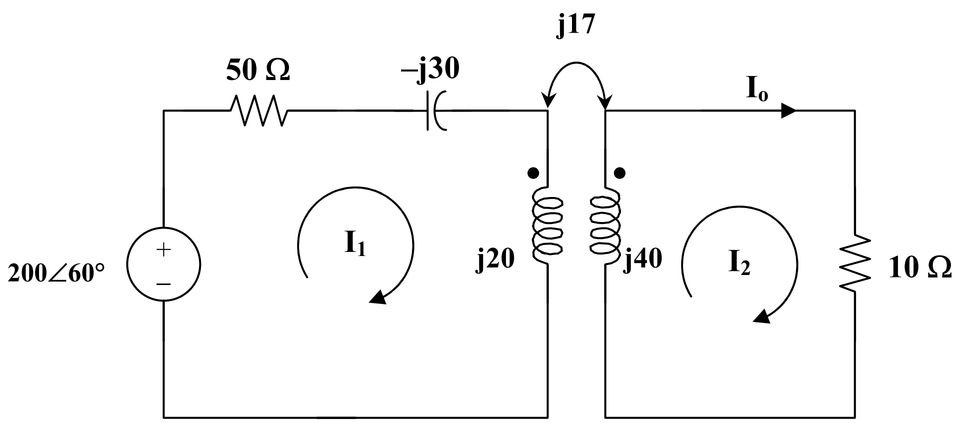
i_o = **<u>2.2sin(2t – 4.88°) A</u>**

Chapter 13, Solution 26.

$$M = k\sqrt{L_1L_2}$$

$$\omega M = k\sqrt{\omega L_1 \omega L_2} = 0.6\sqrt{20x40} = 17$$

The frequency-domain equivalent circuit is shown below.



For mesh 1,

$$200\angle 60° = (50 - j30 + j20)I_1 + j17I_2 = (50 - j10)I_1 + j17I_2 \qquad (1)$$

For mesh 2,

$$0 = (10 + j40)I_2 + j17I_1 \qquad (2)$$

In matrix form,
$$\begin{bmatrix} 200\angle 60° \\ 0 \end{bmatrix} = \begin{bmatrix} 50 - j10 & j17 \\ j17 & 10 + j40 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$\Delta = 900 + j100$, $\Delta_1 = 2000\angle 60°(1 + j4) = 8246.2\angle 136°$, $\Delta_2 = 3400\angle{-30°}$

$I_2 = \Delta_2/\Delta = 3.755\angle{-36.34°}$

$I_o = I_2 =$ **$\underline{3.755\angle{-36.34°}\text{ A}}$**

Switching the dot on the winding on the right only reverses the direction of I_o. This can be seen by looking at the resulting value of Δ_2 which now becomes $3400\angle 150°$. Thus,

$I_o =$ **$\underline{3.755\angle 143.66°\text{ A}}$**

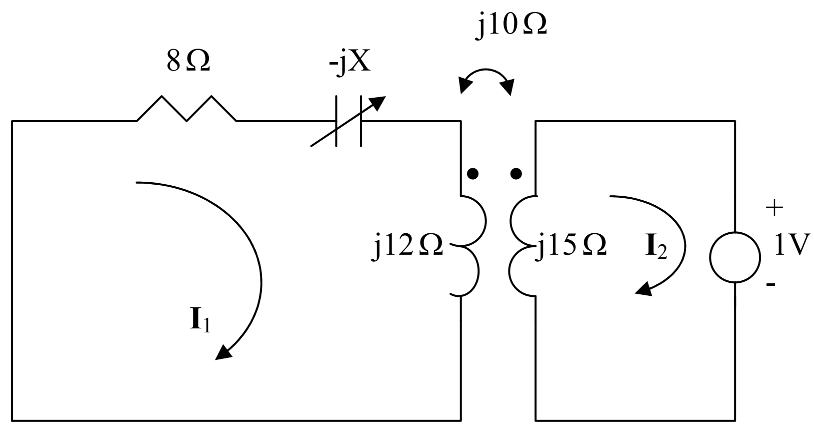
Chapter 13, Solution 27.

Z_{in} = –j4 + j5 + 9/(12 + j6) = 0.6 + j.07 = 0.922∠49.4°

I_1 = 12∠0°/0.922∠49.4° = **<u>13∠–49.4° A</u>**

Chapter 13, Solution 28.

We find Z_{Th} by replacing the 20-ohm load with a unit source as shown below.



For mesh 1, $0 = (8 - jX + j12)I_1 - j10I_2$ (1)

For mesh 2, $1 + j15I_2 - j10I_1 = 0 \longrightarrow I_1 = 1.5I_2 - 0.1j$ (2)

Substituting (2) into (1) leads to

$$I_2 = \frac{-1.2 + j0.8 + 0.1X}{12 + j8 - j1.5X}$$

$$Z_{Th} = \frac{1}{-I_2} = \frac{12 + j8 - j1.5X}{1.2 - j0.8 - 0.1X}$$

$$|Z_{Th}| = 20 = \frac{\sqrt{12^2 + (8 - 1.5X)^2}}{\sqrt{(1.2 - 0.1X)^2 + 0.8^2}} \longrightarrow 0 = 1.75X^2 + 72X - 624$$

Solving the quadratic equation yields **<u>X = 6.425</u>**

Chapter 13, Solution 29.

30 mH becomes $j\omega L = j30x10^{-3}x10^{3} = j30$

50 mH becomes j50

Let $X = \omega M$

Using the concept of reflected impedance,

$Z_{in} = 10 + j30 + X^2/(20 + j50)$

$I_1 = V/Z_{in} = 165/(10 + j30 + X^2/(20 + j50))$

$p = 0.5|I_1|^2(10) = 320$ leads to $|I_1|^2 = 64$ or $|I_1| = 8$

$8 = |165(20 + j50)/(X^2 + (10 + j30)(20 + j50))|$

$= |165(20 + j50)/(X^2 - 1300 + j1100)|$

or $64 = 27225(400 + 2500)/((X^2 - 1300)^2 + 1{,}210{,}000)$

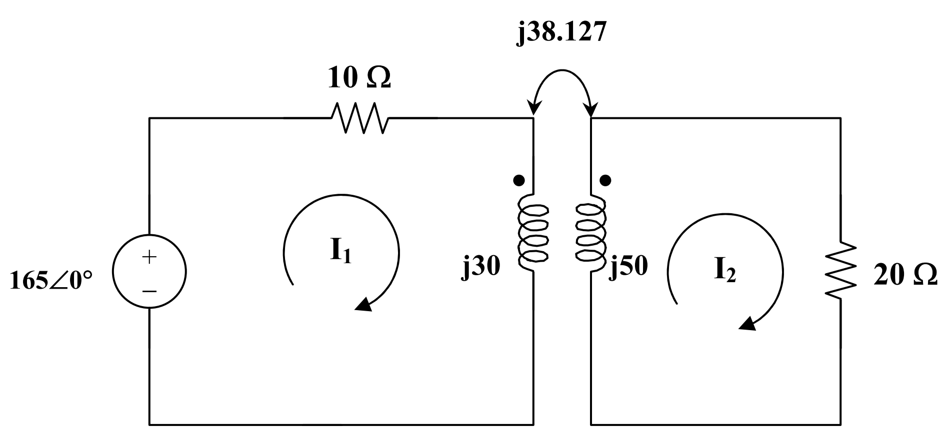
$(X^2 - 1300)^2 + 1{,}210{,}000 = 1{,}233{,}633$

$X = 33.86$ or 38.13

If $X = 38.127 = \omega M$

$M = 38.127$ mH

$k = M/\sqrt{L_1L_2} = 38.127/\sqrt{30x50} = \underline{\mathbf{0.984}}$



$165 = (10 + j30)I_1 - j38.127I_2$ (1)

$0 = (20 + j50)I_2 - j38.127I_1$ (2)

In matrix form, $\begin{bmatrix}165\\0\end{bmatrix} = \begin{bmatrix}10 + j30 & -j38.127\\ -j38.127 & 20 + j50\end{bmatrix}\begin{bmatrix}I_1\\I_2\end{bmatrix}$

$\Delta = 154 + j1100 = 1110.73\angle 82.03°, \ \Delta_1 = 888.5\angle 68.2°, \ \Delta_2 = j6291$

$I_1 = \Delta_1/\Delta = 8\angle -13.81°, \ I_2 = \Delta_2/\Delta = 5.664\angle 7.97°$

$i_1 = 8\cos(1000t - 13.83°), \ i_2 = 5.664\cos(1000t + 7.97°)$

At $t = 1.5$ ms, $1000t = 1.5$ rad $= 85.94°$

$i_1 = 8\cos(85.94° - 13.83°) = 2.457$

$i_2 = 5.664\cos(85.94° + 7.97°) = -0.3862$

$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 + Mi_1i_2$

$= 0.5(30)(2.547)^2 + 0.5(50)(-0.3862)^2 - 38.127(2.547)(-0.3862)$

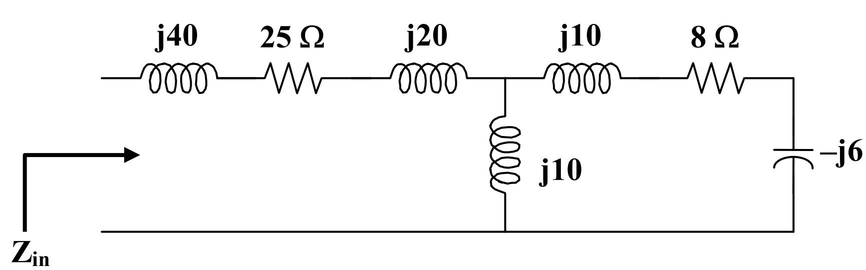
$=$ **130.51 mJ**

Chapter 13, Solution 30.

(a) $Z_{in} = j40 + 25 + j30 + (10)^2/(8 + j20 - j6)$
$= 25 + j70 + 100/(8 + j14) =$ **(28.08 + j64.62) ohms**

(b) $j\omega L_a = j30 - j10 = j20, \ j\omega L_b = j20 - j10 = j10, \ j\omega L_c = j10$

Thus the Thevenin Equivalent of the linear transformer is shown below.



$Z_{in} = j40 + 25 + j20 + j10||(8 + j4) = 25 + j60 + j10(8 + j4)/(8 + j14)$

$=$ **(28.08 + j64.62) ohms**

Chapter 13, Solution 31.

(a) $L_a = L_1 - M = \underline{\mathbf{10\ H}}$

$L_b = L_2 - M = \underline{\mathbf{15\ H}}$

$L_c = M = \underline{\mathbf{5\ H}}$

(b) $L_1L_2 - M^2 = 300 - 25 = 275$

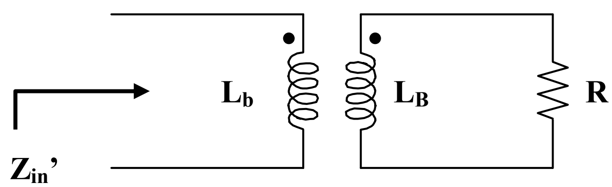
$L_A = (L_1L_2 - M^2)/(L_1 - M) = 275/15 = \underline{\mathbf{18.33\ H}}$

$L_B = (L_1L_2 - M^2)/(L_1 - M) = 275/10 = \underline{\mathbf{27.5\ H}}$

$L_C = (L_1L_2 - M^2)/M = 275/5 = \underline{\mathbf{55\ H}}$

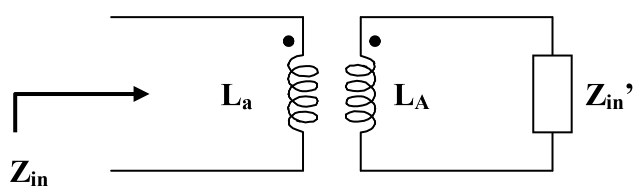
Chapter 13, Solution 32.

We first find Z_{in} for the second stage using the concept of reflected impedance.



$Z_{in}' = j\omega L_b + \omega^2 M_b^2/(R + j\omega L_b) = (j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)/(R + j\omega L_b)$ (1)

For the first stage, we have the circuit below.



$Z_{in} = j\omega L_a + \omega^2 M_a^2/(j\omega L_a + Z_{in})$

$= (-\omega^2 L_a^2 + \omega^2 M_a^2 + j\omega L_a Z_{in})/(j\omega L_a + Z_{in})$ (2)

Substituting (1) into (2) gives,

$$= \frac{-\omega^2 L_a^2 + \omega^2 M_a^2 + j\omega L_a \dfrac{(j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)}{R + j\omega L_b}}{j\omega L_a + \dfrac{j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2}{R + j\omega L_b}}$$

$$= \frac{-R\omega^2 L_a^2 + \omega^2 M_a^2 R - j\omega^3 L_b L_a + j\omega^3 L_b M_a^2 + j\omega L_a(j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)}{j\omega R La - \omega^2 L_a L_b + j\omega L_b R - \omega^2 L_a^2 + \omega^2 M_b^2}$$

$$Z_{in} = \underline{\frac{\boldsymbol{\omega^2 R(L_a^2 + L_a L_b - M_a^2) + j\omega^3(L_a^2 L_b + L_a L_b^2 - L_a M_b^2 - L_b M_a^2)}}{\boldsymbol{\omega^2(L_a L_b + L_b^2 - M_b^2) - j\omega R(L_a + L_b)}}}$$

Chapter 13, Solution 33.

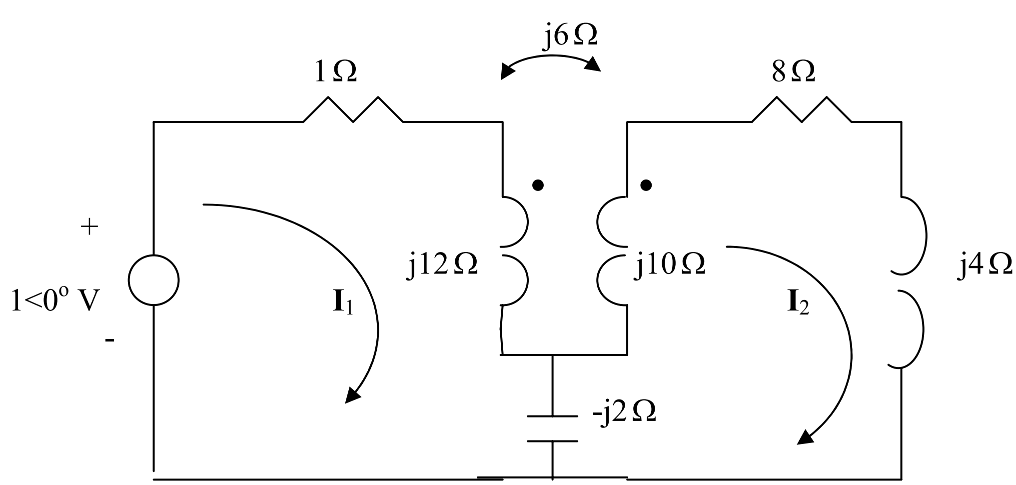
$Z_{in} = 10 + j12 + (15)^2/(20 + j\,40 - j5) = 10 + j12 + 225/(20 + j35)$

$= 10 + j12 + 225(20 - j35)/(400 + 1225)$

$=$ $\underline{(12.769 + j7.154)\ \text{ohms}}$

Chapter 13, Solution 34.

Insert a 1-V voltage source at the input as shown below.



For loop 1,

$$1 = (1 + j10)I_1 - j4I_2 \qquad (1)$$

For loop 2,

$$0 = (8 + j4 + j10 - j2)I_2 + j2I_1 - j6I_1 \quad \longrightarrow \quad 0 = -jI_1 + (2 + j3)I_2 \qquad (2)$$

Solving (1) and (2) leads to $\mathbf{I}_1$=0.019 –j0.1068

$$Z = \frac{1}{I_1} = 1.6154 + j9.077 = \underline{9.219\angle 79.91^o\,\Omega}$$

Alternatively, an easier way to obtain **Z** is to replace the transformer with its equivalent T circuit and use series/parallel impedance combinations. This leads to exactly the same result.

Chapter 13, Solution 35.

For mesh 1, $16 = (10 + j4)I_1 + j2I_2$ (1)

For mesh 2, $0 = j2I_1 + (30 + j26)I_2 - j12I_3$ (2)

For mesh 3, $0 = -j12I_2 + (5 + j11)I_3$ (3)

We may use MATLAB to solve (1) to (3) and obtain

$$I_1 = 1.3736 - j0.5385 = \underline{1.4754\angle -21.41^o \text{ A}}$$
$$I_2 = -0.0547 - j0.0549 = \underline{0.0775\angle -134.85^o \text{ A}}$$
$$I_3 = -0.0268 - j0.0721 = \underline{0.077\angle -110.41^o \text{ A}}$$

Chapter 13, Solution 36.

Following the two rules in section 13.5, we obtain the following:

(a) V_2/V_1 = **–n**, I_2/I_1 = **–1/n** (n = V_2/V_1)

(b) V_2/V_1 = **–n**, I_2/I_1 = **–1/n**

(c) V_2/V_1 = **n**, I_2/I_1 = **1/n**

(d) V_2/V_1 = **n**, I_2/I_1 = **–1/n**

Chapter 13, Solution 37.

(a) $n = \frac{V_2}{V_1} = \frac{2400}{480} = \underline{5}$

(b) $S_1 = I_1V_1 = S_2 = I_2V_2 = 50{,}000 \longrightarrow I_1 = \frac{50{,}000}{480} = \underline{104.17\ \text{A}}$

(c) $I_2 = \frac{50{,}000}{2400} = \underline{20.83\ \text{A}}$

Chapter 13, Solution 38.

$Z_{in} = Z_p + Z_L/n^2, \quad n = v_2/v_1 = 230/2300 = 0.1$

$v_2 = 230\ \text{V}, \quad s_2 = v_2I_2^*$

$I_2^* = s_2/v_2 = 17.391\angle{-53.13°}$ or $I_2 = 17.391\angle 53.13°$ A

$Z_L = v_2/I_2 = 230\angle 0°/17.391\angle 53.13° = 13.235\angle{-53.13°}$

$Z_{in} = 2\angle 10° + 1323.5\angle{-53.13°}$

$= 1.97 + j0.3473 + 794.1 - j1058.8$

$Z_{in} =$ $\underline{\mathbf{1.324\angle{-53.05°}\ kohms}}$

Chapter 13, Solution 39.

Referred to the high-voltage side,

$Z_L = (1200/240)^2(0.8\angle 10°) = 20\angle 10°$

$Z_{in} = 60\angle{-30°} + 20\angle 10° = 76.4122\angle{-20.31°}$

$I_1 = 1200/Z_{in} = 1200/76.4122\angle{-20.31°} =$ $\underline{\mathbf{15.7\angle 20.31°\ A}}$

Since $S = I_1v_1 = I_2v_2$, $I_2 = I_1v_1/v_2$

$= (1200/240)(15.7\angle 20.31°) =$ $\underline{\mathbf{78.5\angle 20.31°\ A}}$