



## TEORIA DA PROBABILIDADE

$$\Pr[A \cup B] = \Pr[A] + \Pr[B] - \Pr[A \cap B] \quad \Pr[A | B] = \frac{\Pr[A \cap B]}{\Pr[B]}$$

$\Pr[A \cup B] = \Pr[A] + \Pr[B]$ , para  $A$  e  $B$  eventos disjuntos.

$\Pr[A \cap B] = \Pr[A] \Pr[B]$ , para  $A$  e  $B$  eventos independentes.

$$\Pr[B] = \sum_i \Pr[B | A_i] \Pr[A_i], \quad \text{onde } \{A_i\} \text{ são eventos que particionam o espaço amostral.}$$

## VARIÁVEIS ALEATÓRIAS

PMF  $p_X(x)$  Função massa de probabilidade

PDF  $f_X(x)$  Função densidade de probabilidade

CDF  $F_X(x)$  Função distribuição cumulativa

$$p_X(x) = \Pr[X = x] \quad \Pr[a \leq X \leq b] = \int_{a^-}^{b^+} f_X(x) dx \quad F_X(x) = \Pr[X \leq x] = \int_{-\infty}^{x^+} f_X(u) du$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dy \quad f_{X|Y=y}(x) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

$$f_X(x) = \sum_i f_{X|A_i}(x) \Pr[A_i], \quad \text{onde } \{A_i\} \text{ são eventos que particionam o espaço amostral.}$$

## DISTRIBUIÇÃO NORMAL

$$X \sim N(\mu, \sigma^2) \iff f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{u^2}{2}} du \quad Q(x) = 1 - \Phi(x)$$

$$X \sim N(\mu, \sigma^2) \implies \Pr[X \leq x] = \Phi\left(\frac{x-\mu}{\sigma}\right), \Pr[X > x] = Q\left(\frac{x-\mu}{\sigma}\right)$$

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## VALOR ESPERADO

$$E[g(X)] = \sum_{x \in \mathcal{X}} g(x)p_X(x) \quad E[g(X)] = \int_{-\infty}^{\infty} g(x)f_X(x)dx$$

$$E[g(X, Y)] = \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} g(x, y)p_{X, Y}(x, y) \quad E[g(X, Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y)f_{X, Y}(x, y)dxdy$$

$$\mu_X = E[X] \quad \sigma_X^2 = \text{var}[X] = E[(X - \mu_X)^2] = E[X^2] - E[X]^2$$

$$\text{cov}[X, Y] = E[(X - \mu_X)(Y - \mu_Y)] = E[XY] - E[X]E[Y]$$

$$\text{var}[X + Y] = \text{var}[X] + \text{var}[Y] + 2 \text{cov}[X, Y]$$

$$\rho_{X, Y} = \frac{\text{cov}[X, Y]}{\sqrt{\text{var}[X] \text{var}[Y]}}$$

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## VETORES ALEATÓRIOS

$$\vec{\mu}_{\vec{X}} = E[\vec{X}] = \begin{bmatrix} E[X_1] \\ E[X_2] \\ \vdots \\ E[X_n] \end{bmatrix} \quad K_{\vec{X}} = E[(\vec{X} - \vec{\mu}_{\vec{X}})(\vec{X} - \vec{\mu}_{\vec{X}})^T] = \begin{bmatrix} \text{var}[X_1] & \text{cov}[X_1, X_2] & \cdots & \text{cov}[X_1, X_n] \\ \text{cov}[X_2, X_1] & \text{var}[X_2] & \cdots & \text{cov}[X_2, X_n] \\ \vdots & \vdots & \ddots & \vdots \\ \text{cov}[X_n, X_1] & \text{cov}[X_n, X_2] & \cdots & \text{var}[X_n] \end{bmatrix}$$

$$\vec{Y} = A\vec{X} + \vec{b} \quad \Longrightarrow \quad \vec{\mu}_{\vec{Y}} = A\vec{\mu}_{\vec{X}} + \vec{b}, \quad K_{\vec{Y}} = AK_{\vec{X}}A^T$$

$$\vec{X} \sim N(\vec{\mu}, K) \quad \Longleftrightarrow \quad f_{\vec{X}}(\vec{x}) = \frac{1}{(2\pi)^{n/2} \sqrt{\det K}} \exp\left(-\frac{1}{2}(\vec{x} - \vec{\mu})^T K^{-1}(\vec{x} - \vec{\mu})\right)$$

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## PROCESSOS ESTOCÁSTICOS

$$\mu_X(t) = E[X(t)] \quad R_X(t_1, t_2) = E[X(t_1)X(t_2)] \quad K_X(t_1, t_2) = \text{cov}[X(t_1), X(t_2)]$$

$$K_X(t_1, t_2) = R_X(t_1, t_2) - \mu_X(t_1)\mu_X(t_2)$$

$$S_X(f) = \mathcal{F}\{R_X(\tau)\} \quad \mu_Y = H(0)\mu_X \quad S_Y(f) = |H(f)|^2 S_X(f)$$

$$P_X = E[X^2(t)]$$

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## SÉRIES

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}, \quad \sum_{k=0}^{\infty} kr^k = \frac{r}{(1-r)^2}, \quad \sum_{k=0}^{\infty} k^2 r^k = \frac{r(1+r)}{(1-r)^3}. \quad (|r| < 1)$$

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## IDENTIDADES TRIGONOMÉTRICAS

$$\cos(a \pm b) = \cos a \cos b \mp \operatorname{sen} a \operatorname{sen} b \quad \operatorname{sen}(a \pm b) = \operatorname{sen} a \cos b \mp \operatorname{sen} b \cos a$$

$$\cos a \cos b = \frac{1}{2}[\cos(a - b) + \cos(a + b)] \quad \operatorname{sen} a \operatorname{sen} b = \frac{1}{2}[\cos(a - b) - \cos(a + b)]$$

$$\cos^2 a = \frac{1}{2}(1 + \cos 2a) \quad \operatorname{sen}^2 a = \frac{1}{2}(1 - \cos 2a)$$

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## FÓRMULA DE EULER

$$e^{j\theta} = \cos \theta + j \operatorname{sen} \theta \quad \cos \theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta}) \quad \operatorname{sen} \theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$$

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## SINAIS BÁSICOS

$$\operatorname{rect}(x) = \begin{cases} 1, & \text{se } |x| < 1/2, \\ 0, & \text{caso contrário.} \end{cases} \quad \operatorname{tri}(x) = \begin{cases} 1 - |x|, & \text{se } |x| < 1, \\ 0, & \text{caso contrário.} \end{cases} \quad \operatorname{sinc}(x) = \frac{\operatorname{sen}(\pi x)}{\pi x}$$

$$\operatorname{sign}(x) = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0. \end{cases} \quad u(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0. \end{cases}$$

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## DEFINIÇÃO E PROPRIEDADES DA TRANSFORMADA DE FOURIER

$$X(f) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft} dt \quad x(t) = \mathcal{F}^{-1}\{X(f)\} = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft} df$$

$$\mathcal{F}\{ax_1(t) + bx_2(t)\} = a\mathcal{F}\{x_1(t)\} + b\mathcal{F}\{x_2(t)\} \quad \mathcal{F}\{x(at)\} = \frac{1}{|a|} X\left(\frac{f}{a}\right)$$

$$\mathcal{F}\{x(t - t_0)\} = \mathcal{F}\{x(t)\}e^{-j2\pi t_0 f} \quad \mathcal{F}\{x(t)e^{j2\pi f_0 t}\} = X(f - f_0)$$

$$\mathcal{F}\{x_1(t)x_2(t)\} = \mathcal{F}\{x_1(t)\} \star \mathcal{F}\{x_2(t)\} \quad \mathcal{F}\{x_1(t) \star x_2(t)\} = \mathcal{F}\{x_1(t)\}\mathcal{F}\{x_2(t)\}$$

$$\mathcal{F}\{X(t)\} = x(-f)$$

$$\mathcal{F}\left\{\frac{d^n}{dt^n}x(t)\right\} = (j2\pi f)^n X(f) \quad \mathcal{F}\{t^n x(t)\} = \left(\frac{j}{2\pi}\right)^n \frac{d^n}{df^n} X(f)$$

## PARES TRANSFORMADOS DE FOURIER

Sejam  $t_0$ ,  $f_0$  e  $T$  constantes reais positivas.

$x(t)$	$X(f)$
1	$\delta(f)$
$\delta(t)$	1
$e^{j2\pi f_0 t}$	$\delta(f - f_0)$
$\delta(t - t_0)$	$e^{-j2\pi t_0 f}$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$
$\text{sen}(2\pi f_0 t)$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$
$\text{rect}(t/t_0)$	$t_0 \text{sinc}(t_0 f)$
$\text{sinc}(t/t_0)$	$t_0 \text{rect}(t_0 f)$
$\text{tri}(t/t_0)$	$t_0 \text{sinc}^2(t_0 f)$
$\text{sinc}^2(t/t_0)$	$t_0 \text{tri}(t_0 f)$
$\text{sign}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{t}$	$-j\pi \text{sign}(f)$
$u(t)$	$\frac{1}{2} \left( \frac{1}{j\pi f} + \delta(f) \right)$
$J_0(t)$	$\frac{2 \text{rect}(\pi f)}{\sqrt{1 - (2\pi f)^2}}$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{1}{T} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T}\right)$
$e^{-t/t_0} u(t)$	$\frac{t_0}{1 + j2\pi t_0 f}$
$e^{- t/t_0 }$	$\frac{2t_0}{1 + (2\pi t_0 f)^2}$
$e^{-\frac{1}{2}(t/t_0)^2}$	$t_0 \sqrt{2\pi} e^{-\frac{1}{2}(2\pi t_0 f)^2}$