INSTITUTO FEDERAL DE SANTA CATARINA
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ENGENHARIA DE TELECOMUNICAÇÕES

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TEORIA DA PROBABILIDADE

$$\Pr[A \cup B] = \Pr[A] + \Pr[B] - \Pr[A \cap B] \qquad \Pr[A \mid B] = \frac{\Pr[A \cap B]}{\Pr[B]}$$

$$\Pr[A \cup B] = \Pr[A] + \Pr[B], \quad \text{para } A \text{ e } B \text{ eventos disjuntos.}$$

$$\Pr[A \cap B] = \Pr[A]\Pr[B], \quad \text{para } A \text{ e } B \text{ eventos independentes.}$$

$$\Pr[B] = \sum_i \Pr[B \mid A_i]\Pr[A_i], \quad \text{onde } \{A_i\} \text{ são eventos que particionam o espaço amostral.}$$

VARIÁVEIS ALEATÓRIAS

PMF	$p_X(x)$	Função massa de probabilidade
PDF	$f_X(x)$	Função densidade de probabilidade
CDF	$F_X(x)$	Função distribuição cumulativa

$$p_X(x) = \Pr[X = x] \qquad \Pr[a \leq X \leq b] = \int_{a^-}^{b^+} f_X(x)dx \qquad F_X(x) = \Pr[X \leq x] = \int_{-\infty}^{x^+} f_X(u)du$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dy \qquad f_{X|Y=y}(x) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

$$f_X(x) = \sum_i f_{X|A_i}(x)\Pr[A_i], \quad \text{onde } \{A_i\} \text{ são eventos que particionam o espaço amostral.}$$

DISTRIBUIÇÕES IMPORTANTES

$$X \sim \mathrm{N}(\mu, \sigma^2) \iff f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$X \sim \mathrm{Exp}(\lambda) \iff f_X(x) = \frac{1}{\lambda} \mathrm{e}^{-\frac{x}{\lambda}} \mathrm{u}(x)$$

$$X \sim \mathrm{N}(\mu, \sigma^2) \implies \Pr[X \leq x] = \Phi\left(\frac{x-\mu}{\sigma}\right), \ \Pr[X > x] = Q\left(\frac{x-\mu}{\sigma}\right)$$

VALOR ESPERADO

$$\mathrm{E}[g(X)] = \sum_{x \in \mathcal{X}} g(x) p_{\mathrm{X}}(x) \qquad \mathrm{E}[g(X)] = \int_{-\infty}^{\infty} g(x) f_{\mathrm{X}}(x) dx$$

$$\mu_X = \mathrm{E}[X] \qquad \sigma_X^2 = \mathrm{var}[X] = \mathrm{E}[(X - \mu_X)^2] = \mathrm{E}[X^2] - \mathrm{E}[X]^2$$

$$\mathrm{cov}[X, Y] = \mathrm{E}[(X - \mu_X)(Y - \mu_Y)] = \mathrm{E}[XY] - \mathrm{E}[X]\mathrm{E}[Y]$$

$$\mathrm{var}[X + Y] = \mathrm{var}[X] + \mathrm{var}[Y] + 2\,\mathrm{cov}[X, Y]$$

$$\rho_{X,Y} = \frac{\mathrm{cov}[X, Y]}{\sqrt{\mathrm{var}[X]\,\mathrm{var}[Y]}}$$

VETORES ALEATÓRIOS

$$\vec{\mu}_{\vec{X}} = \mathrm{E}[\vec{X}] = \begin{bmatrix} \mathrm{E}[X_1] \\ \mathrm{E}[X_2] \\ \vdots \\ \mathrm{E}[X_n] \end{bmatrix} \qquad K_{\vec{X}} = \mathrm{E}[(\vec{X} - \vec{\mu}_{\vec{Y}})(\vec{X} - \vec{\mu}_{\vec{Y}})^{\mathrm{T}}] = \begin{bmatrix} \mathrm{var}[X_1] & \mathrm{cov}[X_1, X_2] & \cdots & \mathrm{cov}[X_1, X_n] \\ \mathrm{cov}[X_2, X_1] & \mathrm{var}[X_2] & \cdots & \mathrm{cov}[X_2, X_n] \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}[X_n, X_1] & \mathrm{cov}[X_n, X_2] & \cdots & \mathrm{var}[X_n] \end{bmatrix}$$

$$\vec{Y} = A\vec{X} + \vec{b} \quad \Longrightarrow \quad \vec{\mu}_{\vec{Y}} = A\vec{\mu}_{\vec{X}} + \vec{b}, \quad K_{\vec{Y}} = AK_{\vec{X}}A^{\mathrm{T}}$$

$$\vec{X} \sim \mathrm{N}(\vec{\mu}, K) \quad \Longleftrightarrow \quad f_{\vec{X}}(\vec{x}) = \frac{1}{(2\pi)^{n/2}\sqrt{\det K}} \exp\left(-\frac{1}{2}(\vec{x} - \vec{\mu})^{\mathrm{T}} K^{-1} (\vec{x} - \vec{\mu})\right)$$

PROCESSOS ESTOCÁSTICOS

$$\mu_X(t) = \mathrm{E}[X(t)] \qquad R_X(t_1, t_2) = \mathrm{E}[X(t_1)X(t_2)] \qquad K_X(t_1, t_2) = \mathrm{cov}[X(t_1), X(t_2)]$$

$$K_X(t_1, t_2) = R_X(t_1, t_2) - \mu_X(t_1)\mu_X(t_2)$$

$$S_X(f) = \mathcal{F}\{R_X(\tau)\} \qquad \mu_Y = H(0)\mu_X \qquad S_Y(f) = |H(f)|^2 S_X(f)$$

$$P_X = \mathrm{E}[X^2(t)]$$

SÉRIES

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1 - r}, \qquad \sum_{k=0}^{\infty} kr^k = \frac{r}{(1 - r)^2}, \qquad \sum_{k=0}^{\infty} k^2 r^k = \frac{r(1 + r)}{(1 - r)^3}. \qquad (|r| < 1)$$

IDENTIDADES TRIGONOMÉTRICAS

$$\cos(a \pm b) = \cos a \cos b \mp \operatorname{sen} a \operatorname{sen} b \qquad \operatorname{sen}(a \pm b) = \operatorname{sen} a \cos b \mp \operatorname{sen} b \cos a$$

$$\cos a \cos b = \frac{1}{2}[\cos(a-b) + \cos(a+b)] \qquad \operatorname{sen} a \operatorname{sen} b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$$

$$\cos^2 a = \frac{1}{2}(1 + \cos 2a) \qquad \operatorname{sen}^2 a = \frac{1}{2}(1 - \cos 2a)$$

FÓRMULA DE EULER

$$\mathrm{e}^{\mathrm{j}\theta} = \cos\theta + \mathrm{j}\operatorname{sen}\theta \qquad \cos\theta = \frac{1}{2}(\mathrm{e}^{\mathrm{j}\theta} + \mathrm{e}^{-\mathrm{j}\theta}) \qquad \operatorname{sen}\theta = \frac{1}{2\mathrm{j}}(\mathrm{e}^{\mathrm{j}\theta} - \mathrm{e}^{-\mathrm{j}\theta})$$

SINAIS BÁSICOS

$$\operatorname{rect}(x) = \begin{cases} 1, & \text{se } |x| < 1/2, \\ 0, & \text{caso contrário.} \end{cases} \qquad \operatorname{tri}(x) = \begin{cases} 1 - |x|, & \text{se } |x| < 1, \\ 0, & \text{caso contrário.} \end{cases} \qquad \operatorname{sinc}(x) = \frac{\operatorname{sen}(\pi x)}{\pi x}$$

$$\operatorname{sign}(x) = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0. \end{cases} \qquad \mathrm{u}(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0. \end{cases}$$

DEFINIÇÃO E PROPRIEDADES DA TRANSFORMADA DE FOURIER

$$X(f) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-\mathrm{j}2\pi f t}dt \qquad x(t) = \mathcal{F}^{-1}\{X(f)\} = \int_{-\infty}^{\infty} X(f)e^{\mathrm{j}2\pi t f}df$$

$$\mathcal{F}\{ax_1(t) + bx_2(t)\} = a\mathcal{F}\{x_1(t)\} + b\mathcal{F}\{x_2(t)\} \qquad \mathcal{F}\{x(at)\} = \frac{1}{|a|}X\left(\frac{f}{a}\right)$$

$$\mathcal{F}\{x(t-t_0)\} = \mathcal{F}\{x(t)\}\mathrm{e}^{-\mathrm{j}2\pi t_0 f} \qquad \mathcal{F}\{x(t)\mathrm{e}^{\mathrm{j}2\pi f_0 t}\} = X(f - f_0)$$

$$\mathcal{F}\{x_1(t)x_2(t)\} = \mathcal{F}\{x_1(t)\} \star \mathcal{F}\{x_2(t)\} \qquad \mathcal{F}\{x_1(t) \star x_2(t)\} = \mathcal{F}\{x_1(t)\}\mathcal{F}\{x_2(t)\}$$

$$\mathcal{F}\{X(t)\} = x(-f)$$

$$\mathcal{F}\left\{\frac{d^n}{dt^n}x(t)\right\} = (\mathrm{j}2\pi f)^n X(f) \qquad \mathcal{F}\{t^n x(t)\} = \left(\frac{\mathrm{j}}{2\pi}\right)^n \frac{d^n}{df^n}X(f)$$

PARES TRANSFORMADOS DE FOURIER

Sejam t_0, f_0 e T constantes reais positivas.

$x(t)$	$X(f)$
1	$\delta(f)$
$\delta(t)$	1
$\mathrm{e}^{\mathrm{j}2\pi f_0 t}$	$\delta(f - f_0)$
$\delta(t - t_0)$	$\mathrm{e}^{-\mathrm{j}2\pi t_0 f}$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$
$\mathrm{sen}(2\pi f_0 t)$	$\frac{1}{2\mathrm{j}}\delta(f - f_0) - \frac{1}{2\mathrm{j}}\delta(f + f_0)$
$\mathrm{rect}(t/t_0)$	$t_0\,\mathrm{sinc}(t_0 f)$
$\mathrm{sinc}(t/t_0)$	$t_0\,\mathrm{rect}(t_0 f)$
$\mathrm{tri}(t/t_0)$	$t_0\,\mathrm{sinc}^2(t_0 f)$
$\mathrm{sinc}^2(t/t_0)$	$t_0\,\mathrm{tri}(t_0 f)$
$\mathrm{sign}(t)$	$\frac{1}{\mathrm{j}\pi f}$
$\frac{1}{t}$	$-\mathrm{j}\pi\,\mathrm{sign}(f)$
$\mathrm{u}(t)$	$\frac{1}{2}\left(\frac{1}{\mathrm{j}\pi f} + \delta(f)\right)$
$\mathrm{J}_0(t)$	$\frac{2\,\mathrm{rect}(\pi f)}{\sqrt{1 - (2\pi f)^2}}$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{1}{T}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T}\right)$
$\mathrm{e}^{-t/t_0}\mathrm{u}(t)$	$\frac{t_0}{1 + \mathrm{j}2\pi t_0 f}$
$\mathrm{e}^{-\lvert t/t_0\rvert}$	$\frac{2t_0}{1 + (2\pi t_0 f)^2}$
$\mathrm{e}^{-\frac{1}{2}(t/t_0)^2}$	$t_0\sqrt{2\pi}\mathrm{e}^{-\frac{1}{2}(2\pi t_0 f)^2}$