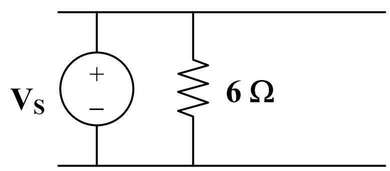
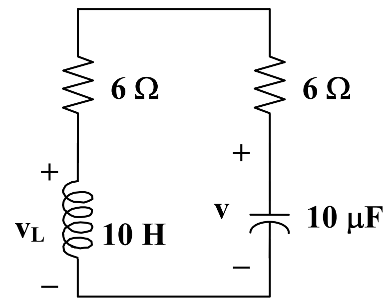
Chapter 8, Solution 1.

(a) At t = 0-, the circuit has reached steady state so that the equivalent circuit is shown in Figure (a).



(a)



(b)

$$i(0-) = 12/6 = 2A, \quad v(0-) = 12V$$

At t = 0+, $i(0+) = i(0-) =$ **2A**, $v(0+) = v(0-) =$ **12V**

(b) For t > 0, we have the equivalent circuit shown in Figure (b).

$$v_L = Ldi/dt \text{ or } di/dt = v_L/L$$

Applying KVL at t = 0+, we obtain,

$$v_L(0+) - v(0+) + 10i(0+) = 0$$

$$v_L(0+) - 12 + 20 = 0, \text{ or } v_L(0+) = -8$$

Hence, $di(0+)/dt = -8/2 =$ **-4 A/s**

Similarly, $i_C = Cdv/dt$, or $dv/dt = i_C/C$

$$i_C(0+) = -i(0+) = -2$$

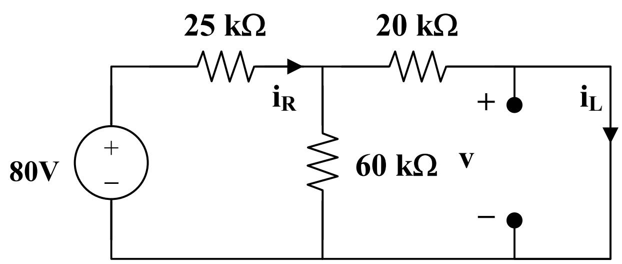
$dv(0+)/dt = -2/0.4 =$ **-5 V/s**

(c) As t approaches infinity, the circuit reaches steady state.

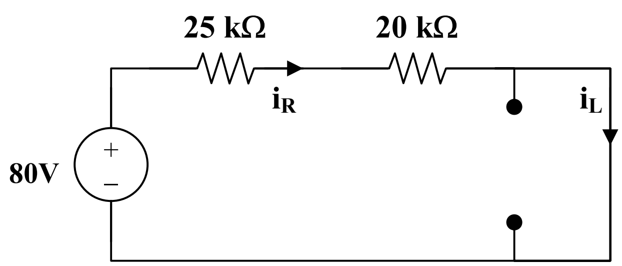
$i(\infty) =$ **0 A**, $v(\infty) =$ **0 V**

Chapter 8, Solution 2.

(a) At t = 0-, the equivalent circuit is shown in Figure (a).



(a)



(b)

$$60||20 = 15 \text{ kohms}, \; i_R(0-) = 80/(25+15) = 2\text{mA}.$$

By the current division principle,

$$i_L(0-) = 60(2\text{mA})/(60+20) = 1.5 \text{ mA}$$

$$v_C(0-) = 0$$

At t = 0+,

$$v_C(0+) = v_C(0-) = 0$$

$$i_L(0+) = i_L(0-) = \underline{\mathbf{1.5\ mA}}$$

$$80 = i_R(0+)(25+20) + v_C(0-)$$

$$i_R(0+) = 80/45\text{k} = \underline{\mathbf{1.778\ mA}}$$

But, $i_R = i_C + i_L$

$$1.778 = i_C(0+) + 1.5 \text{ or } i_C(0+) = \underline{\mathbf{0.278\ mA}}$$

(b) $v_L(0+) = v_C(0+) = 0$

But, $v_L = Ldi_L/dt$ and $di_L(0+)/dt = v_L(0+)/L = 0$

$di_L(0+)/dt =$ **<u>0</u>**

Again, $80 = 45i_R + v_C$

$0 = 45di_R/dt + dv_C/dt$

But, $dv_C(0+)/dt = i_C(0+)/C = 0.278$ mohms/1 μF = 278 V/s

Hence, $di_R(0+)/dt = (-1/45)dv_C(0+)/dt = -278/45$

$di_R(0+)/dt =$ **<u>-6.1778 A/s</u>**

Also, $i_R = i_C + i_L$

$di_R(0+)/dt = di_C(0+)/dt + di_L(0+)/dt$

$-6.1788 = di_C(0+)/dt + 0$, or $di_C(0+)/dt =$ **<u>-6.1788 A/s</u>**

(c) As t approaches infinity, we have the equivalent circuit in Figure (b).

$i_R(\infty) = i_L(\infty) = 80/45k =$ **<u>1.778 mA</u>**

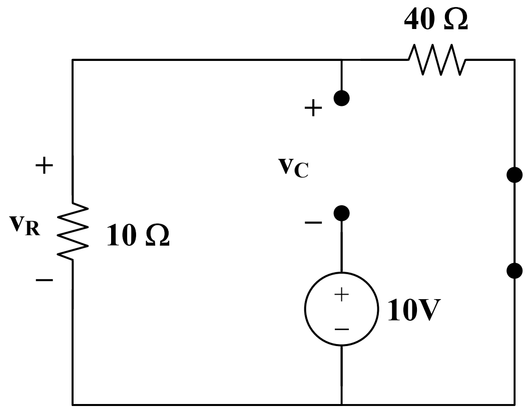
$i_C(\infty) = Cdv(\infty)/dt =$ **<u>0</u>**.

Chapter 8, Solution 3.

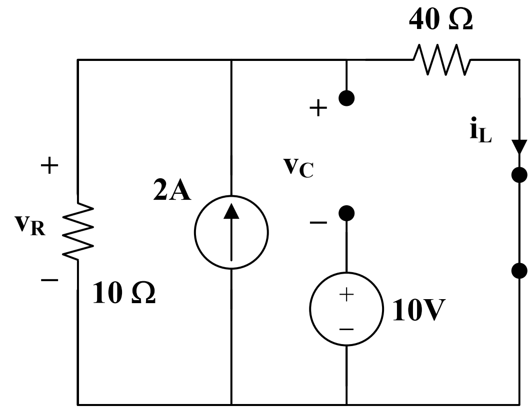
At $t = 0^-$, $u(t) = 0$. Consider the circuit shown in Figure (a). $i_L(0^-) = 0$, and $v_R(0^-) = 0$. But, $-v_R(0^-) + v_C(0^-) + 10 = 0$, or $v_C(0^-) = -10V$.

(a) At $t = 0^+$, since the inductor current and capacitor voltage cannot change abruptly, the inductor current must still be equal to **<u>0A</u>**, the capacitor has a voltage equal to **<u>–10V</u>**. Since it is in series with the +10V source, together they represent a direct short at $t = 0^+$. This means that the entire 2A from the current source flows through the capacitor and not the resistor. Therefore, $v_R(0^+) =$ **<u>0 V</u>**.

(b) At $t = 0^+$, $v_L(0+) = 0$, therefore $Ldi_L(0+)/dt = v_L(0^+) = 0$, thus, $di_L/dt =$ **<u>0A/s</u>**, $i_C(0^+) = 2$ A, this means that $dv_C(0^+)/dt = 2/C =$ **<u>8 V/s</u>**. Now for the value of $dv_R(0^+)/dt$. Since $v_R = v_C + 10$, then $dv_R(0^+)/dt = dv_C(0^+)/dt + 0 =$ **<u>8 V/s</u>**.



(a)



(b)

(c) As t approaches infinity, we end up with the equivalent circuit shown in Figure (b).

$$i_L(\infty) = 10(2)/(40 + 10) = \underline{\textbf{400 mA}}$$

$$v_C(\infty) = 2[10||40] - 10 = 16 - 10 = \underline{\textbf{6V}}$$

$$v_R(\infty) = 2[10||40] = \underline{\textbf{16 V}}$$

Chapter 8, Solution 4.

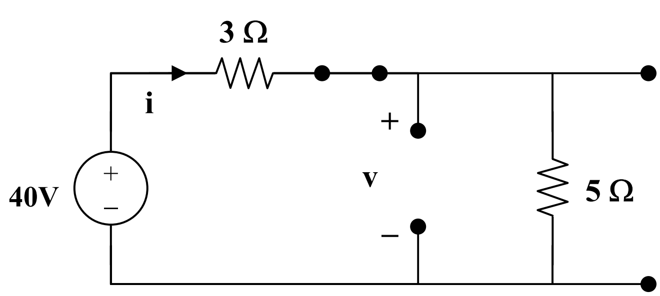
(a) At $t = 0^-$, $u(-t) = 1$ and $u(t) = 0$ so that the equivalent circuit is shown in Figure (a).

$$i(0^-) = 40/(3 + 5) = 5A, \text{ and } v(0^-) = 5i(0^-) = 25V.$$

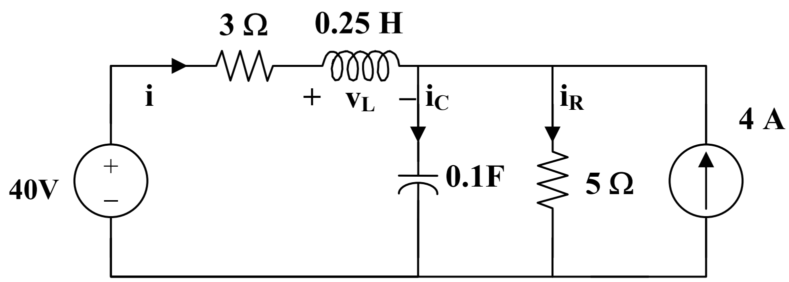
Hence,

$$i(0^+) = i(0^-) = \underline{\textbf{5A}}$$

$$v(0^+) = v(0^-) = \underline{\textbf{25V}}$$



(a)



(b)

(b) $i_C = Cdv/dt$ or $dv(0^+)/dt = i_C(0^+)/C$

For $t = 0^+$, $4u(t) = 4$ and $4u(-t) = 0$. The equivalent circuit is shown in Figure (b). Since i and v cannot change abruptly,

$$i_R = v/5 = 25/5 = 5A, \ i(0^+) + 4 = i_C(0^+) + i_R(0^+)$$

$$5 + 4 = i_C(0^+) + 5 \text{ which leads to } i_C(0^+) = 4$$

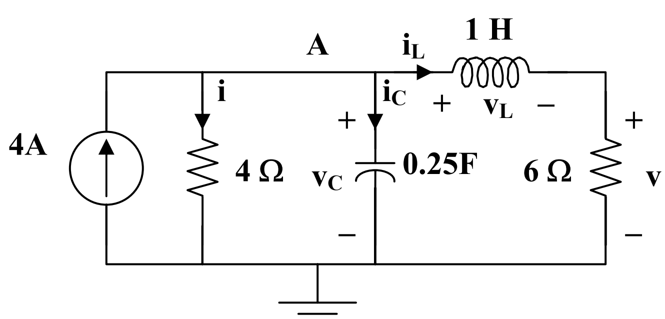
$$dv(0^+)/dt = 4/0.1 = \underline{\mathbf{40\ V/s}}$$

Chapter 8, Solution 5.

(a) For $t < 0$, $4u(t) = 0$ so that the circuit is not active (all initial conditions = 0).

$$i_L(0-) = 0 \text{ and } v_C(0-) = 0.$$

For $t = 0+$, $4u(t) = 4$. Consider the circuit below.



Since the 4-ohm resistor is in parallel with the capacitor,

$$i(0+) = v_C(0+)/4 = 0/4 = \underline{\mathbf{0\ A}}$$

Also, since the 6-ohm resistor is in series with the inductor,
$v(0+) = 6i_L(0+) = \underline{\mathbf{0V}}$.

(b) $di(0+)/dt = d(v_R(0+)/R)/dt = (1/R)dv_R(0+)/dt = (1/R)dv_C(0+)/dt$

$$= (1/4)4/0.25 \text{ A/s} = \underline{\mathbf{4\ A/s}}$$

$v = 6i_L$ or $dv/dt = 6di_L/dt$ and $dv(0+)/dt = 6di_L(0+)/dt = 6v_L(0+)/L = 0$

Therefore $dv(0+)/dt = \underline{\mathbf{0\ V/s}}$

(c) As t approaches infinity, the circuit is in steady-state.

$$i(\infty) = 6(4)/10 = \underline{\mathbf{2.4\ A}}$$

$$v(\infty) = 6(4 - 2.4) = \underline{\mathbf{9.6\ V}}$$

Chapter 8, Solution 6.

(a) Let i = the inductor current. For $t < 0$, $u(t) = 0$ so that

$$i(0) = 0 \text{ and } v(0) = 0.$$

For $t > 0$, $u(t) = 1$. Since, $v(0+) = v(0-) = 0$, and $i(0+) = i(0-) = 0$.

$$v_R(0+) = Ri(0+) = \underline{\mathbf{0\ V}}$$

Also, since $v(0+) = v_R(0+) + v_L(0+) = 0 = 0 + v_L(0+)$ or $v_L(0+) = \underline{\mathbf{0\ V}}$.
(1)

(b) Since $i(0+) = 0$, $i_C(0+) = V_S/R_S$

But, $i_C = Cdv/dt$ which leads to $dv(0+)/dt = V_S/(CR_S)$ (2)

From (1), $dv(0+)/dt = dv_R(0+)/dt + dv_L(0+)/dt$ (3)

$v_R = iR$ or $dv_R/dt = Rdi/dt$ (4)

But, $v_L = Ldi/dt$, $v_L(0+) = 0 = Ldi(0+)/dt$ and $di(0+)/dt = 0$ (5)

From (4) and (5), $dv_R(0+)/dt = \underline{\mathbf{0\ V/s}}$

From (2) and (3), $dv_L(0+)/dt = dv(0+)/dt = \underline{\mathbf{V_s/(CR_s)}}$

(c) As t approaches infinity, the capacitor acts like an open circuit, while the inductor acts like a short circuit.

$$v_R(\infty) = \underline{\mathbf{[R/(R + R_s)]V_s}}$$

$$v_L(\infty) = \underline{\mathbf{0\ V}}$$

Chapter 8, Solution 7.

$$s^2 + 4s + 4 = 0, \text{ thus } s_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4x4}}{2} = -2, \text{ repeated roots.}$$

$$v(t) = [(A + Bt)e^{-2t}], \; v(0) = 1 = A$$

$$dv/dt = [Be^{-2t}] + [-2(A + Bt)e^{-2t}]$$

$$dv(0)/dt = -1 = B - 2A = B - 2 \text{ or } B = 1.$$

Therefore, $v(t)$ = **[(1 + t)e^{-2t}] V**

Chapter 8, Solution 8.

$s^2 + 6s + 9 = 0$, thus $s_{1,2} = \frac{-6 \pm \sqrt{6^2 - 36}}{2} = -3$, repeated roots.

$$i(t) = [(A + Bt)e^{-3t}], \; i(0) = 0 = A$$

$$di/dt = [Be^{-3t}] + [-3(Bt)e^{-3t}]$$

$$di(0)/dt = 4 = B.$$

Therefore, $i(t)$ = **[4te^{-3t}] A**

Chapter 8, Solution 9.

$$s^2 + 10s + 25 = 0, \text{ thus } s_{1,2} = \frac{-10 \pm \sqrt{10 - 10}}{2} = -5, \text{ repeated roots.}$$

$$i(t) = [(A + Bt)e^{-5t}], \; i(0) = 10 = A$$

$$di/dt = [Be^{-5t}] + [-5(A + Bt)e^{-5t}]$$

$$di(0)/dt = 0 = B - 5A = B - 50 \text{ or } B = 50.$$

Therefore, $i(t)$ = **[(10 + 50t)e^{-5t}] A**

Chapter 8, Solution 10.

$$s^2 + 5s + 4 = 0, \text{ thus } s_{1,2} = \frac{-5 \pm \sqrt{25-16}}{2} = -4, -1.$$

$$v(t) = (Ae^{-4t} + Be^{-t}), \ v(0) = 0 = A + B, \text{ or } B = -A$$

$$dv/dt = (-4Ae^{-4t} - Be^{-t})$$

$$dv(0)/dt = 10 = -4A - B = -3A \text{ or } A = -10/3 \text{ and } B = 10/3.$$

Therefore, $v(t) =$ **$(-(10/3)e^{-4t} + (10/3)e^{-t})$ V**

Chapter 8, Solution 11.

$$s^2 + 2s + 1 = 0, \text{ thus } s_{1,2} = \frac{-2 \pm \sqrt{4-4}}{2} = -1, \text{ repeated roots.}$$

$$v(t) = [(A + Bt)e^{-t}], \ v(0) = 10 = A$$

$$dv/dt = [Be^{-t}] + [-(A + Bt)e^{-t}]$$

$$dv(0)/dt = 0 = B - A = B - 10 \text{ or } B = 10.$$

Therefore, $v(t) =$ **$[(10 + 10t)e^{-t}]$ V**

Chapter 8, Solution 12.

(a) Overdamped when $C > 4L/(R^2) = 4x0.6/400 = 6x10^{-3}$, or $C >$ **6 mF**

(b) Critically damped when $C =$ **6 mF**

(c) Underdamped when $C <$ **6mF**

Chapter 8, Solution 13.

Let R||60 = R_o. For a series RLC circuit,

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.01x4}} = 5$$

For critical damping, $\omega_o = \alpha = R_o/(2L) = 5$

or $R_o = 10L = 40 = 60R/(60 + R)$

which leads to R = **<u>120 ohms</u>**

Chapter 8, Solution 14.

This is a series, source-free circuit. 60||30 = 20 ohms

$$\alpha = R/(2L) = 20/(2x2) = 5 \text{ and } \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.04}} = 5$$

$\omega_o = \alpha$ leads to critical damping

$$i(t) = [(A + Bt)e^{-5t}],\ i(0) = 2 = A$$

$$v = Ldi/dt = 2\{[Be^{-5t}] + [-5(A + Bt)e^{-5t}]\}$$

$$v(0) = 6 = 2B - 10A = 2B - 20 \text{ or } B = 13.$$

Therefore, i(t) = $\underline{\mathbf{[(2 + 13t)e^{-5t}]\ A}}$

Chapter 8, Solution 15.

This is a series, source-free circuit. 60||30 = 20 ohms

$$\alpha = R/(2L) = 20/(2x2) = 5 \text{ and } \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.04}} = 5$$

$\omega_o = \alpha$ leads to critical damping

$$i(t) = [(A + Bt)e^{-5t}],\ i(0) = 2 = A$$

$$v = Ldi/dt = 2\{[Be^{-5t}] + [-5(A + Bt)e^{-5t}]\}$$

$$v(0) = 6 = 2B - 10A = 2B - 20 \text{ or } B = 13.$$

Therefore, i(t) = $\underline{\mathbf{[(2 + 13t)e^{-5t}]\ A}}$

Chapter 8, Solution 16.

At $t = 0$, $i(0) = 0$, $v_C(0) = 40x30/50 = 24V$

For $t > 0$, we have a source-free RLC circuit.

$$\alpha = R/(2L) = (40 + 60)/5 = 20 \text{ and } \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} x2.5}} = 20$$

$\omega_o = \alpha$ leads to critical damping

$$i(t) = [(A + Bt)e^{-20t}],\ i(0) = 0 = A$$

$$di/dt = \{[Be^{-20t}] + [-20(Bt)e^{-20t}]\},$$

but $di(0)/dt = -(1/L)[Ri(0) + v_C(0)] = -(1/2.5)[0 + 24]$

Hence, $B = -9.6$ or $i(t) =$ **$[-9.6te^{-20t}]$ A**

Chapter 8, Solution 17.

$$i(0) = I_0 = 0,\ v(0) = V_0 = 4x15 = 60$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) = -4(0 + 60) = -240$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}\frac{1}{25}}} = 10$$

$$\alpha = \frac{R}{2L} = \frac{10}{2\frac{1}{4}} = 20, \text{ which is} > \omega_o.$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -20 \pm \sqrt{300} = -20 \pm 10\sqrt{3} = -2.68, -37.32$$

$$i(t) = A_1e^{-2.68t} + A_2e^{-37.32t}$$

$$i(0) = 0 = A_1 + A_2,\ \frac{di(0)}{dt} = -2.68A_1 - 37.32A_2 = -240$$

This leads to $A_1 = -6.928 = -A_2$

$$i(t) = 6.928\left(e^{-37.32t} - e^{-268t}\right)$$

Since, $v(t) = \frac{1}{C}\int_0^t i(t)dt + 60$, we get

$v(t) =$ **$(60 + 64.53e^{-2.68t} - 4.6412e^{-37.32t})$ V**

Chapter 8, Solution 18.

When the switch is off, we have a source-free parallel RLC circuit.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25x1}} = 2, \qquad \alpha = \frac{1}{2RC} = 0.5$$

$\alpha < \omega_o \quad \longrightarrow \quad$ underdamped case $\quad \omega_d = \sqrt{\omega_o{}^2 - \alpha^2} = \sqrt{4 - 0.25} = 1.936$

$I_o(0) = i(0) =$ initial inductor current = 20/5 = 4A

$V_o(0) = v(0) =$ initial capacitor voltage = 0 V

$$v(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t) = e^{-0.5\alpha t}(A_1 \cos 1.936t + A_2 \sin 1.936t)$$

v(0) =0 = A_1

$$\frac{dv}{dt} = e^{-0.5\alpha t}(-0.5)(A_1 \cos 1.936t + A_2 \sin 1.936t) + e^{-0.5\alpha t}(-1.936A_1 \sin 1.936t + 1.936A_2 \cos 1.936t)$$

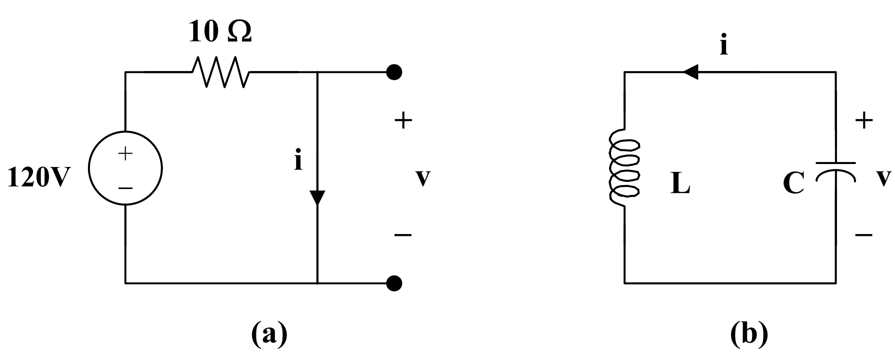
$$\frac{dv(0)}{dt} = -\frac{(V_o + RI_o)}{RC} = -\frac{(0+4)}{1} = -4 = -0.5A_1 + 1.936A_2 \quad \longrightarrow \quad A_2 = -2.066$$

Thus,

$$\underline{v(t) = -2.066e^{-0.5t} \sin 1.936t}$$

Chapter 8, Solution 19.

For t < 0, the equivalent circuit is shown in Figure (a).



i(0) = 120/10 = 12, v(0) = 0

For $t > 0$, we have a series RLC circuit as shown in Figure (b) with $R = 0 = \alpha$.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4}} = 0.5 = \omega_d$$

$$i(t) = [A\cos 0.5t + B\sin 0.5t], \quad i(0) = 12 = A$$

$$v = -Ldi/dt, \text{ and } -v/L = di/dt = 0.5[-12\sin 0.5t + B\cos 0.5t],$$

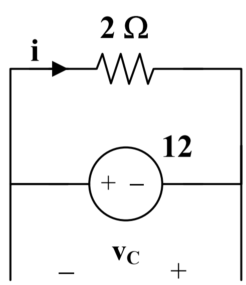
which leads to $-v(0)/L = 0 = B$

Hence, $i(t) = 12\cos 0.5t$ A and $v = 0.5$

However, $v = -Ldi/dt = -4(0.5)[-12\sin 0.5t] =$ **<u>24sin0.5t V</u>**

Chapter 8, Solution 20.

For $t < 0$, the equivalent circuit is as shown below.



$$v(0) = -12V \text{ and } i(0) = 12/2 = 6A$$

For $t > 0$, we have a series RLC circuit.

$$\alpha = R/(2L) = 2/(2x0.5) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{0.5x1/4} = 2\sqrt{2}$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{8-4} = 2$$

$$i(t) = (A\cos 2t + B\sin 2t)e^{-2t}$$

$$i(0) = 6 = A$$

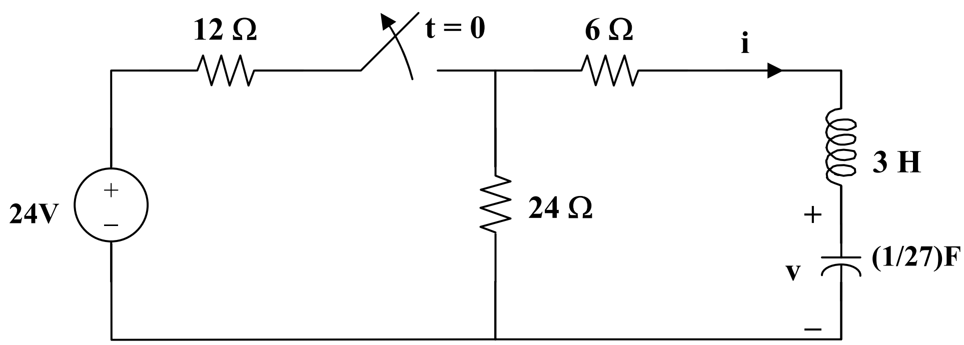
$$di/dt = -2(6\cos 2t + B\sin 2t)e^{-2t} + (-2x6\sin 2t + 2B\cos 2t)e^{-\alpha t}$$

$$di(0)/dt = -12 + 2B = -(1/L)[Ri(0) + v_C(0)] = -2[12 - 12] = 0$$

Thus, B = 6 and i(t) = $\underline{\mathbf{(6\cos 2t + 6\sin 2t)e^{-2t}\ A}}$

Chapter 8, Solution 21.

By combining some resistors, the circuit is equivalent to that shown below.
60||(15 + 25) = 24 ohms.



At t = 0-, i(0) = 0, v(0) = 24x24/36 = 16V

For t > 0, we have a series RLC circuit. R = 30 ohms, L = 3 H, C = (1/27) F

$$\alpha = R/(2L) = 30/6 = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{3x1/27} = 3, \text{ clearly } \alpha > \omega_o \text{ (overdamped response)}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm \sqrt{5^2 - 3^2} = -9, -1$$

$$v(t) = [Ae^{-t} + Be^{-9t}],\ v(0) = 16 = A + B \qquad (1)$$

$$i = Cdv/dt = C[-Ae^{-t} - 9Be^{-9t}]$$

$$i(0) = 0 = C[-A - 9B] \text{ or } A = -9B \qquad (2)$$

From (1) and (2), B = -2 and A = 18.

Hence, v(t) = $\underline{\mathbf{(18e^{-t} - 2e^{-9t})\ V}}$

Chapter 8, Solution 22.

$$\alpha = 20 = 1/(2RC) \text{ or } RC = 1/40 \quad (1)$$

$$\omega_d = 50 = \sqrt{\omega_o^2 - \alpha^2} \quad \text{which leads to } 2500 + 400 = \omega_o^2 = 1/(LC)$$

$$\text{Thus,} \quad LC \;\; 1/2900 \quad (2)$$

In a parallel circuit, $v_C = v_L = v_R$

But, $i_C = Cdv_C/dt$ or $i_C/C = dv_C/dt$

$$= -80e^{-20t}\cos 50t - 200e^{-20t}\sin 50t + 200e^{-20t}\sin 50t - 500e^{-20t}\cos 50t$$
$$= -580e^{-20t}\cos 50t$$

$i_C(0)/C = -580$ which leads to $C = -6.5\text{x}10^{-3}/(-580) =$ **11.21 μF**

$R = 1/(40C) = 10^6/(2900\text{x}11.21) =$ **2.23 kohms**

$L = 1/(2900\text{x}11.21) =$ **30.76 H**

Chapter 8, Solution 23.

Let $C_o = C + 0.01$. For a parallel RLC circuit,

$$\alpha = 1/(2RC_o), \quad \omega_o = 1/\sqrt{LC_o}$$

$\alpha = 1 = 1/(2RC_o)$, we then have $C_o = 1/(2R) = 1/20 = 50$ mF

$$\omega_o = 1/\sqrt{0.5\text{x}0.5} = 6.32 > \alpha \text{ (underdamped)}$$

$C_o = C + 10$ mF $= 50$ mF or **40 mF**

Chapter 8, Solution 24.

For $t < 0$, $u(-t)$ 1, namely, the switch is on.

$v(0) = 0$, $i(0) = 25/5 = 5$A

For $t > 0$, the voltage source is off and we have a source-free parallel RLC circuit.

$$\alpha = 1/(2RC) = 1/(2\text{x}5\text{x}10^{-3}) = 100$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{0.1x10^{-3}} = 100$$

$$\omega_o = \alpha \text{ (critically damped)}$$

$$v(t) = [(A_1 + A_2t)e^{-100t}]$$

$$v(0) = 0 = A_1$$

$$dv(0)/dt = -[v(0) + Ri(0)]/(RC) = -[0 + 5x5]/(5x10^{-3}) = -5000$$

$$\text{But,} \quad dv/dt = [(A_2 + (-100)A_2t)e^{-100t}]$$

$$\text{Therefore,} \quad dv(0)/dt = -5000 = A_2 - 0$$

$$v(t) = \underline{\mathbf{-5000te^{-100t}\ V}}$$

Chapter 8, Solution 25.

In the circuit in Fig. 8.76, calculate $i_o(t)$ and $v_o(t)$ for t>0.

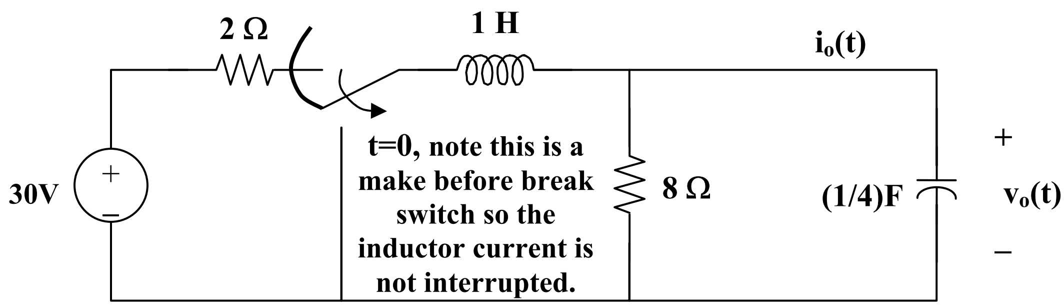


Figure 8.78 For Problem 8.25.

At t = 0^-, $v_o(0) = (8/(2 + 8)(30) = 24$

For t > 0, we have a source-free parallel RLC circuit.

$$\alpha = 1/(2RC) = ¼$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/4} = 2$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{4 - (1/16)} = 1.9843$$

$$v_o(t) = (A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t}$$

$v_o(0) = 24 = A_1$ and $i_o(t) = C(dv_o/dt) = 0$ when $t = 0$.

$$dv_o/dt = -\alpha(A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t} + (-\omega_d A_1\sin\omega_d t + \omega_d A_2\cos\omega_d t)e^{-\alpha t}$$

at $t = 0$, we get $dv_o(0)/dt = 0 = -\alpha A_1 + \omega_d A_2$

Thus, $A_2 = (\alpha/\omega_d)A_1 = (1/4)(24)/1.9843 = 3.024$

$v_o(t) =$ **$(24\cos\omega_d t + 3.024\sin\omega_d t)e^{-t/4}$ volts**

Chapter 8, Solution 26.

$$s^2 + 2s + 5 = 0, \text{ which leads to } s_{1,2} = \frac{-2 \pm \sqrt{4 - 20}}{2} = -1 \pm j4$$

$$i(t) = I_s + [(A_1\cos 4t + A_2\sin 4t)e^{-t}], \quad I_s = 10/5 = 2$$

$$i(0) = 2 = = 2 + A_1, \text{ or } A_1 = 0$$

$$di/dt = [(A_2\cos 4t)e^{-t}] + [(-A_2\sin 4t)e^{-t}] = 4 = 4A_2, \text{ or } A_2 = 1$$

$i(t) =$ **$2 + \sin 4te^{-t}$ A**

Chapter 8, Solution 27.

$$s^2 + 4s + 8 = 0 \text{ leads to } s = \frac{-4 \pm \sqrt{16 - 32}}{2} = -2 \pm j2$$

$$v(t) = V_s + (A_1\cos 2t + A_2\sin 2t)e^{-2t}$$

$8V_s = 24$ means that $V_s = 3$

$v(0) = 0 = 3 + A_1$ leads to $A_1 = -3$

$$dv/dt = -2(A_1\cos 2t + A_2\sin 2t)e^{-2t} + (-2A_1\sin 2t + 2A_2\cos 2t)e^{-2t}$$

$0 = dv(0)/dt = -2A_1 + 2A_2$ or $A_2 = A_1 = -3$

$v(t) =$ **$[3 - 3(\cos 2t + \sin 2t)e^{-2t}]$ volts**

Chapter 8, Solution 28.

The characteristic equation is $s^2 + 6s + 8$ with roots

$$s_{1,2} = \frac{-6 \pm \sqrt{36-32}}{2} = -4,-2$$

Hence,

$$i(t) = I_s + Ae^{-2t} + Be^{-4t}$$

$$8I_s = 12 \quad \longrightarrow \quad I_s = 1.5$$

$$i(0) = 0 \quad \longrightarrow \quad 0 = 1.5 + A + B \qquad (1)$$

$$\frac{di}{dt} = -2Ae^{-2t} - 4Be^{-4t}$$

$$\frac{di(0)}{dt} = 2 = -2A - 4B \quad \longrightarrow \quad 0 = 1 + A + 2B \qquad (2)$$

Solving (1) and (2) leads to A=-2 and B=0.5.

$$i(t) = \underline{1.5 - 2e^{-2t} + 0.5e^{-4t}} \text{ A}$$

Chapter 8, Solution 29.

(a) $s^2 + 4 = 0$ which leads to $s_{1,2} = \pm j2$ (an undamped circuit)

$$v(t) = V_s + A\cos 2t + B\sin 2t$$

$$4V_s = 12 \text{ or } V_s = 3$$

$$v(0) = 0 = 3 + A \text{ or } A = -3$$

$$dv/dt = -2A\sin 2t + 2B\cos 2t$$

dv(0)/dt = 2 = 2B or B = 1, therefore v(t) = **(3 – 3cos2t + sin2t) V**

(b) $s^2 + 5s + 4 = 0$ which leads to $s_{1,2} = -1, -4$

$$i(t) = (I_s + Ae^{-t} + Be^{-4t})$$

$$4I_s = 8 \text{ or } I_s = 2$$

$$i(0) = -1 = 2 + A + B, \text{ or } A + B = -3 \qquad (1)$$

$$di/dt = -Ae^{-t} - 4Be^{-4t}$$

$$di(0)/dt = 0 = -A - 4B, \text{ or } B = -A/4 \qquad (2)$$

From (1) and (2) we get A = -4 and B = 1

$$i(t) = \underline{\mathbf{(2 - 4e^{-t} + e^{-4t})\ A}}$$

(c) $s^2 + 2s + 1 = 0, \ s_{1,2} = -1, -1$

$$v(t) = [V_s + (A + Bt)e^{-t}], \ V_s = 3.$$

$$v(0) = 5 = 3 + A \text{ or } A = 2$$

$$dv/dt = [-(A + Bt)e^{-t}] + [Be^{-t}]$$

$$dv(0)/dt = -A + B = 1 \text{ or } B = 2 + 1 = 3$$

$$v(t) = \underline{\mathbf{[3 + (2 + 3t)e^{-t}]\ V}}$$

Chapter 8, Solution 30.

$$s_1 = -500 = -\alpha + \sqrt{\alpha^2 - \omega_o^2}, \qquad s_2 = -800 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$$

$$s_1 + s_2 = -1300 = -2\alpha \quad \longrightarrow \quad \alpha = 650 = \frac{R}{2L}$$

Hence,

$$L = \frac{R}{2\alpha} = \frac{200}{2x650} = \underline{153.8 \text{ mH}}$$

$$s_1 - s_2 = 300 = 2\sqrt{\alpha^2 - \omega_o^2} \quad \longrightarrow \quad \omega_o = 623.45 = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{(632.45)^2 L} = \underline{16.25 \mu\text{F}}$$

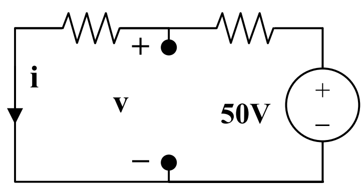
Chapter 8, Solution 31.

For t = 0-, we have the equivalent circuit in Figure (a). For t = 0+, the equivalent circuit is shown in Figure (b). By KVL,

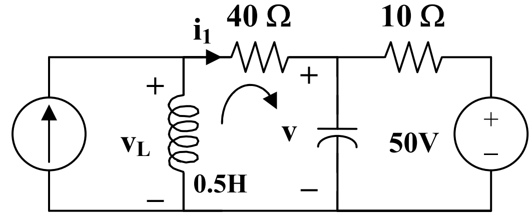
$$v(0+) = v(0-) = 40, \; i(0+) = i(0-) = 1$$

By KCL, 2 = i(0+) + i_1 = 1 + i_1 which leads to i_1 = 1. By KVL, $-v_L + 40i_1 + v(0+) = 0$ which leads to $v_L(0+) = 40x1 + 40 = 80$

$$v_L(0+) = \underline{\textbf{80 V}}, \quad v_C(0+) = \underline{\textbf{40 V}}$$



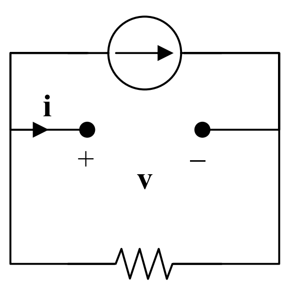
(a)



(b)

Chapter 8, Solution 32.

For t = 0-, the equivalent circuit is shown below.



$$i(0-) = 0, \; v(0-) = -2x6 = -12V$$

For t > 0, we have a series RLC circuit with a step input.

$$\alpha = R/(2L) = 6/2 = 3, \; \omega_o = 1/\sqrt{LC} = 1/\sqrt{0.04}$$

$$s = -3 \pm \sqrt{9-25} = -3 \pm j4$$

$$\text{Thus,} \quad v(t) = V_f + [(A\cos 4t + B\sin 4t)e^{-3t}]$$

$$\text{where } V_f = \text{final capacitor voltage} = 50 \text{ V}$$

$$v(t) = 50 + [(A\cos 4t + B\sin 4t)e^{-3t}]$$

$$v(0) = -12 = 50 + A \text{ which gives } A = -62$$

$$i(0) = 0 = Cdv(0)/dt$$

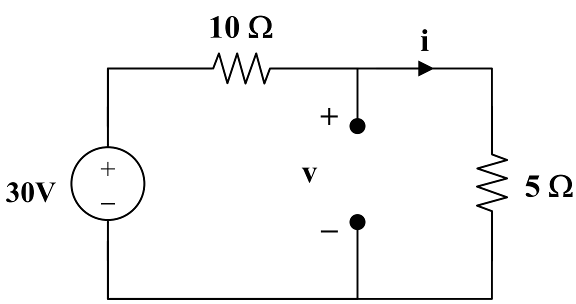
$$dv/dt = [-3(A\cos 4t + B\sin 4t)e^{-3t}] + [4(-A\sin 4t + B\cos 4t)e^{-3t}]$$

$$0 = dv(0)/dt = -3A + 4B \text{ or } B = (3/4)A = -46.5$$

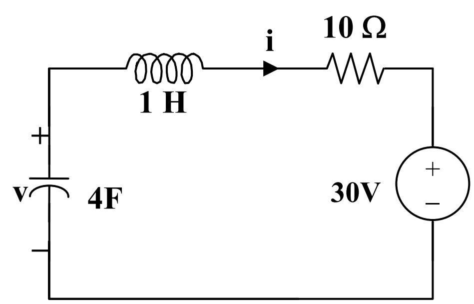
$$v(t) = \underline{\mathbf{\{50 + [(-62\cos 4t - 46.5\sin 4t)e^{-3t}]\}\ V}}$$

Chapter 8, Solution 33.

We may transform the current sources to voltage sources. For $t = 0^-$, the equivalent circuit is shown in Figure (a).



(a)



(b)

$$i(0) = 30/15 = 2 \text{ A}, \ v(0) = 5\text{x}30/15 = 10 \text{ V}$$

For $t > 0$, we have a series RLC circuit.

$$\alpha = R/(2L) = 5/2 = 2.5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{4} = 0.25, \text{ clearly } \alpha > \omega_o \text{ (overdamped response)}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2.5 \pm \sqrt{6.25 - 0.25} = -4.95, -0.05$$

$$v(t) = V_s + [A_1 e^{-4.95t} + A_2 e^{-0.05t}], \ v = 20.$$

$$v(0) = 10 = 20 + A_1 + A_2 \qquad (1)$$

$$i(0) = Cdv(0)/dt \text{ or } dv(0)/dt = 2/4 = 1/2$$

Hence, $\frac{1}{2} = -4.95A_1 - 0.05A_2$ (2)

From (1) and (2), $A_1 = 0, \; A_2 = -10.$

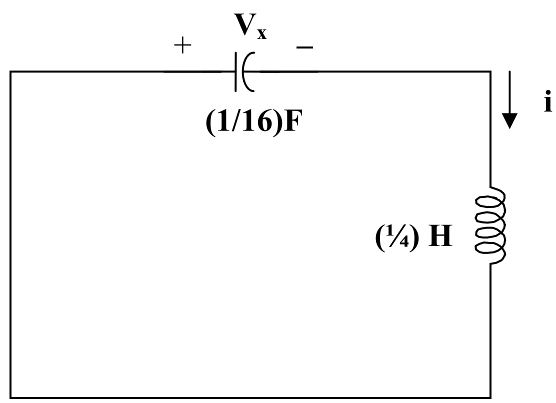
$$v(t) = \underline{\mathbf{\{20 - 10e^{-0.05t}\}\ V}}$$

Chapter 8, Solution 34.

Before t = 0, the capacitor acts like an open circuit while the inductor behaves like a short circuit.

$$i(0) = 0, \; v(0) = 20 \text{ V}$$

For t > 0, the LC circuit is disconnected from the voltage source as shown below.



This is a lossless, source-free, series RLC circuit.

$$\alpha = R/(2L) = 0, \; \omega_o = 1/\sqrt{LC} = 1/\sqrt{\frac{1}{16} + \frac{1}{4}} = 8, \; s = \pm j8$$

Since α is less than ω_o, we have an underdamped response. Therefore,

$$i(t) = A_1\cos 8t + A_2\sin 8t \text{ where } i(0) = 0 = A_1$$

$$di(0)/dt = (1/L)v_L(0) = -(1/L)v(0) = -4x20 = -80$$

However, $di/dt = 8A_2\cos 8t$, thus, $di(0)/dt = -80 = 8A_2$ which leads to $A_2 = -10$

Now we have $i(t) = \underline{\mathbf{-10\sin 8t\ A}}$

Chapter 8, Solution 35.

At t = 0-, $i_L(0) = 0$, $v(0) = v_C(0) = 8$ V

For t > 0, we have a series RLC circuit with a step input.

$$\alpha = R/(2L) = 2/2 = 1,\ \omega_o = 1/\sqrt{LC} = 1/\sqrt{1/5} = \sqrt{5}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1 \pm j2$$

$$v(t) = V_s + [(A\cos 2t + B\sin 2t)e^{-t}],\ V_s = 12.$$

$$v(0) = 8 = 12 + A \text{ or } A = -4,\ i(0) = Cdv(0)/dt = 0.$$

$$\text{But } dv/dt = [-(A\cos 2t + B\sin 2t)e^{-t}] + [2(-A\sin 2t + B\cos 2t)e^{-t}]$$

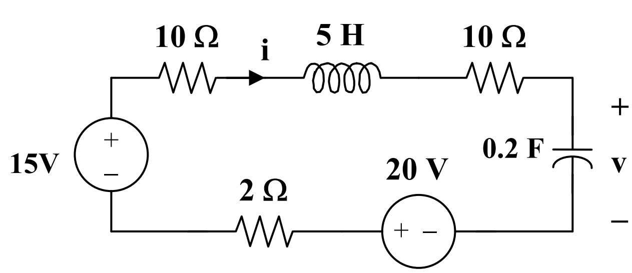
$$0 = dv(0)/dt = -A + 2B \text{ or } 2B = A = -4 \text{ and } B = -2$$

$v(t) =$ <u>**{12 – (4cos2t + 2sin2t)e^{-t} V**</u>.

Chapter 8, Solution 36.

For t = 0-, 3u(t) = 0. Thus, i(0) = 0, and v(0) = 20 V.

For t > 0, we have the series RLC circuit shown below.



$$\alpha = R/(2L) = (2 + 5 + 1)/(2x5) = 0.8$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{5x0.2} = 1$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -0.8 \pm j0.6$$

$$v(t) = V_s + [(A\cos 0.6t + B\sin 0.6t)e^{-0.8t}]$$

$$V_s = 15 + 20 = 35V \text{ and } v(0) = 20 = 35 + A \text{ or } A = -15$$

$$i(0) = Cdv(0)/dt = 0$$

$$\text{But } dv/dt = [-0.8(A\cos 0.6t + B\sin 0.6t)e^{-0.8t}] + [0.6(-A\sin 0.6t + B\cos 0.6t)e^{-0.8t}]$$

$$0 = dv(0)/dt = -0.8A + 0.6B \text{ which leads to } B = 0.8x(-15)/0.6 = -20$$

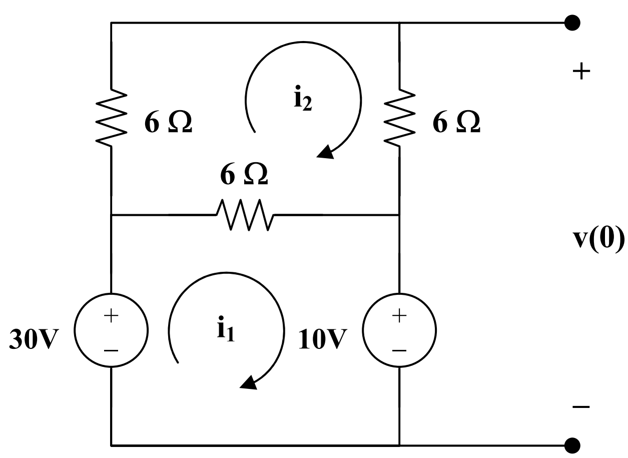
$$v(t) = \underline{\mathbf{\{35 - [(15\cos 0.6t + 20\sin 0.6t)e^{-0.8t}]\}\ V}}$$

$$i = Cdv/dt = 0.2\{[0.8(15\cos 0.6t + 20\sin 0.6t)e^{-0.8t}] + [0.6(15\sin 0.6t - 20\cos 0.6t)e^{-0.8t}]\}$$

$$i(t) = \underline{\mathbf{[(5\sin 0.6t)e^{-0.8t}]\ A}}$$

Chapter 8, Solution 37.

For t = 0-, the equivalent circuit is shown below.



$$18i_2 - 6i_1 = 0 \text{ or } i_1 = 3i_2 \qquad (1)$$

$$-30 + 6(i_1 - i_2) + 10 = 0 \text{ or } i_1 - i_2 = 10/3 \qquad (2)$$

From (1) and (2). $i_1 = 5,\ i_2 = 5/3$

$$i(0) = i_1 = 5A$$

$$-10 - 6i_2 + v(0) = 0$$

$$v(0) = 10 + 6x5/3 = 20$$

For $t > 0$, we have a series RLC circuit.

$$R = 6||12 = 4$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/2)(1/8)} = 4$$

$$\alpha = R/(2L) = (4)/(2x(1/2)) = 4$$

$\alpha = \omega_o$, therefore the circuit is critically damped

$$v(t) = V_s + [(A + Bt)e^{-4t}], \text{ and } V_s = 10$$

$$v(0) = 20 = 10 + A, \text{ or } A = 10$$

$$i = Cdv/dt = -4C[(A + Bt)e^{-4t}] + C[(B)e^{-4t}]$$

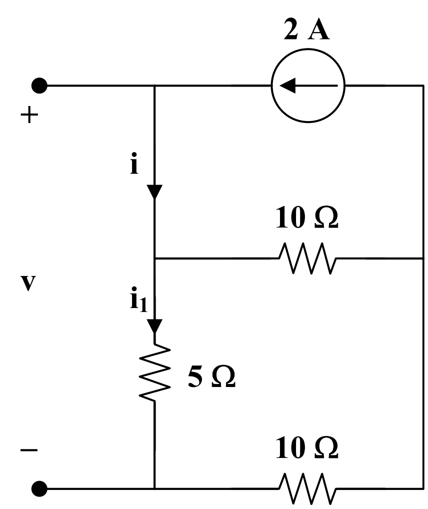
$$i(0) = 5 = C(-4A + B) \text{ which leads to } 40 = -40 + B \text{ or } B = 80$$

$$i(t) = [-(1/2)(10 + 80t)e^{-4t}] + [(10)e^{-4t}]$$

$i(t) =$ **$[(5 - 40t)e^{-4t}]$ A**

Chapter 8, Solution 38.

At $t = 0^-$, the equivalent circuit is as shown.



$$i(0) = 2A, \quad i_1(0) = 10(2)/(10 + 15) = 0.8 \text{ A}$$

$$v(0) = 5i_1(0) = 4V$$

For t > 0, we have a source-free series RLC circuit.

$$R = 5||(10 + 10) = 4 \text{ ohms}$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/3)(3/4)} = 2$$

$$\alpha = R/(2L) = (4)/(2x(3/4)) = 8/3$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -4.431, -0.903$$

$$i(t) = [Ae^{-4.431t} + Be^{-0.903t}]$$

$$i(0) = A + B = 2 \qquad (1)$$

$$di(0)/dt = (1/L)[-Ri(0) + v(0)] = (4/3)(-4x2 + 4) = -16/3 = -5.333$$

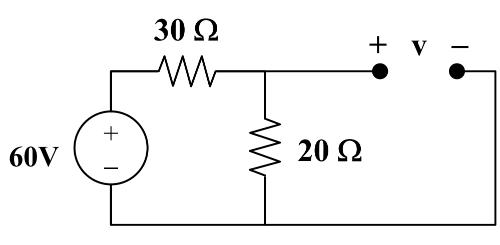
$$\text{Hence, } -5.333 = -4.431A - 0.903B \qquad (2)$$

From (1) and (2), A = 1 and B = 1.

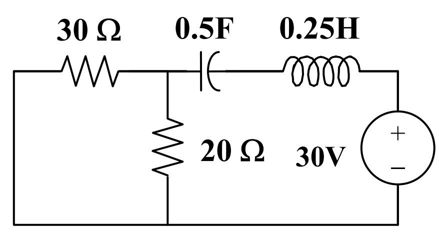
i(t) = $\underline{\mathbf{[e^{-4.431t} + e^{-0.903t}] \text{ A}}}$

Chapter 8, Solution 39.

For t = 0^-, the equivalent circuit is shown in Figure (a). Where 60u(-t) = 60 and 30u(t) = 0.



(a)



(b)

$$v(0) = (20/50)(60) = 24 \text{ and } i(0) = 0$$

For $t > 0$, the circuit is shown in Figure (b).

$$R = 20||30 = 12 \text{ ohms}$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/2)(1/4)} = \sqrt{8}$$

$$\alpha = R/(2L) = (12)/(0.5) = 24$$

Since $\alpha > \omega_o$, we have an overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -47.833, -0.167$$

Thus, $$v(t) = V_s + [Ae^{-47.833t} + Be^{-0.167t}], \quad V_s = 30$$

$$v(0) = 24 = 30 + A + B \text{ or } -6 = A + B \quad (1)$$

$$i(0) = Cdv(0)/dt = 0$$

But, $$dv(0)/dt = -47.833A - 0.167B = 0$$

$$B = -286.43A \quad (2)$$

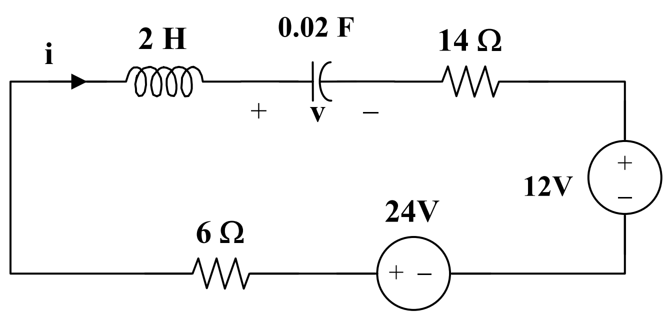
From (1) and (2), $A = 0.021$ and $B = -6.021$

$$v(t) = \underline{\mathbf{30 + [0.021e^{-47.833t} - 6.021e^{-0.167t}]\ V}}$$

Chapter 8, Solution 40.

At $t = 0-$, $v_C(0) = 0$ and $i_L(0) = i(0) = (6/(6 + 2))4 = 3A$

For $t > 0$, we have a series RLC circuit with a step input as shown below.



$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{2x0.02} = 5$$

$$\alpha = R/(2L) = (6 + 14)/(2x2) = 5$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$v(t) = V_s + [(A + Bt)e^{-5t}], \quad V_s = 24 - 12 = 12V$$

$$v(0) = 0 = 12 + A \text{ or } A = -12$$

$$i = Cdv/dt = C\{[Be^{-5t}] + [-5(A + Bt)e^{-5t}]\}$$

$$i(0) = 3 = C[-5A + B] = 0.02[60 + B] \text{ or } B = 90$$

$$\text{Thus,} \quad i(t) = 0.02\{[90e^{-5t}] + [-5(-12 + 90t)e^{-5t}]\}$$

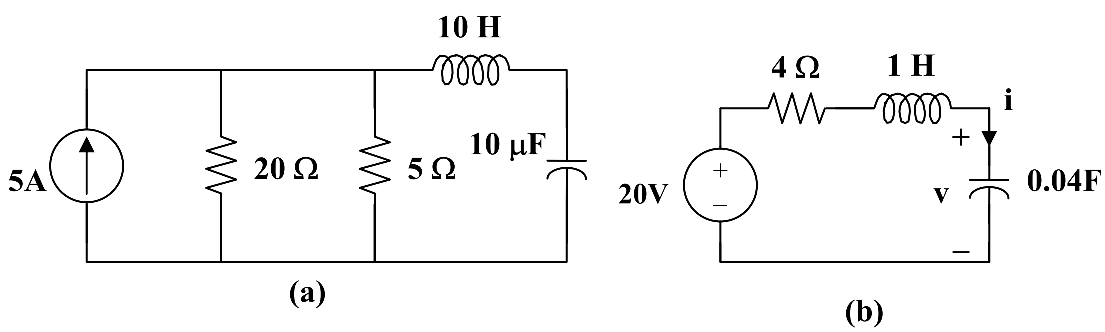
$$i(t) = \underline{\mathbf{\{(3 - 9t)e^{-5t}\}\ A}}$$

Chapter 8, Solution 41.

At t = 0-, the switch is open. i(0) = 0, and

$$v(0) = 5x100/(20 + 5 + 5) = 50/3$$

For t > 0, we have a series RLC circuit shown in Figure (a). After source transformation, it becomes that shown in Figure (b).



$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/25} = 5$$

$$\alpha = R/(2L) = (4)/(2x1) = 2$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2 \pm j4.583$$

Thus, $v(t) = V_s + [(A\cos\omega_d t + B\sin\omega_d t)e^{-2t}]$,

where $\omega_d = 4.583$ and $V_s = 20$

$$v(0) = 50/3 = 20 + A \text{ or } A = -10/3$$

$$i(t) = Cdv/dt = C(-2) [(A\cos\omega_d t + B\sin\omega_d t)e^{-2t}] + C\omega_d[(-A\sin\omega_d t + B\cos\omega_d t)e^{-2t}]$$

$$i(0) = 0 = -2A + \omega_d B$$

$$B = 2A/\omega_d = -20/(3x4.583) = -1.455$$

$$i(t) = C\{[(0\cos\omega_d t + (-2B - \omega_d A)\sin\omega_d t)]e^{-2t}\}$$

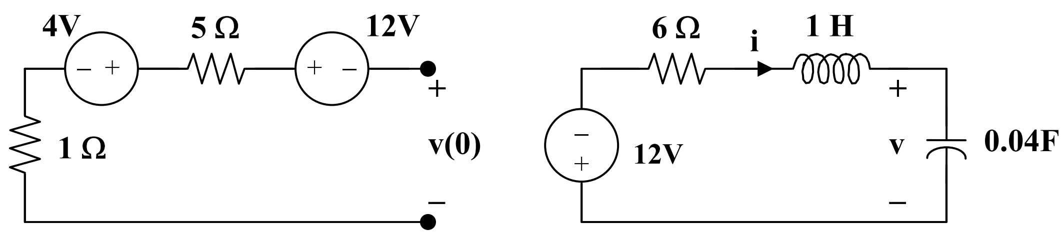
$$= (1/25)\{[(2.91 + 15.2767) \sin\omega_d t)]e^{-2t}\}$$

$$i(t) = \underline{\mathbf{\{0.7275\sin(4.583t)e^{-2t}\}\ A}}$$

Chapter 8, Solution 42.

For t = 0-, we have the equivalent circuit as shown in Figure (a).

$$i(0) = i(0) = 0, \text{ and } v(0) = 4 - 12 = -8V$$



(a) (b)

For t > 0, the circuit becomes that shown in Figure (b) after source transformation.

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/25} = 5$$

$$\alpha = R/(2L) = (6)/(2) = 3$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -3 \pm j4$$

Thus, $v(t) = V_s + [(A\cos 4t + B\sin 4t)e^{-3t}], \quad V_s = -12$

$$v(0) = -8 = -12 + A \text{ or } A = 4$$

$$i = Cdv/dt, \text{ or } i/C = dv/dt = [-3(A\cos 4t + B\sin 4t)e^{-3t}] + [4(-A\sin 4t + B\cos 4t)e^{-3t}]$$

$$i(0) = -3A + 4B \text{ or } B = 3$$

$$v(t) = \underline{\mathbf{\{-12 + [(4\cos 4t + 3\sin 4t)e^{-3t}]\}\ A}}$$

Chapter 8, Solution 43.

For t>0, we have a source-free series RLC circuit.

$$\alpha = \frac{R}{2L} \longrightarrow R = 2\alpha L = 2x8x0.5 = \underline{8\Omega}$$

$$\omega_d = \sqrt{{\omega_o}^2 - \alpha^2} = 30 \longrightarrow \omega_o = \sqrt{900 - 64} = \sqrt{836}$$

$$\omega_o = \frac{1}{\sqrt{LC}} \longrightarrow C = \frac{1}{\omega^2{}_o L} = \frac{1}{836x0.5} = \underline{2.392 \text{ mF}}$$

Chapter 8, Solution 44.

$$\alpha = \frac{R}{2L} = \frac{1000}{2x1} = 500, \quad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100x10^{-9}}} = 10^4$$

$\omega_o > \alpha \longrightarrow$ **<u>underdamped.</u>**

Chapter 8, Solution 45.

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.5} = \sqrt{2}$$

$$\alpha = R/(2L) = (1)/(2x2x0.5) = 0.5$$

Since $\alpha < \omega_o$, we have an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -0.5 \pm j1.323$$

Thus, $$i(t) = I_s + [(A\cos 1.323t + B\sin 1.323t)e^{-0.5t}], \quad I_s = 4$$

$$i(0) = 1 = 4 + A \text{ or } A = -3$$

$$v = v_C = v_L = Ldi(0)/dt = 0$$

$$di/dt = [1.323(-A\sin 1.323t + B\cos 1.323t)e^{-0.5t}] + [-0.5(A\cos 1.323t + B\sin 1.323t)e^{-0.5t}]$$

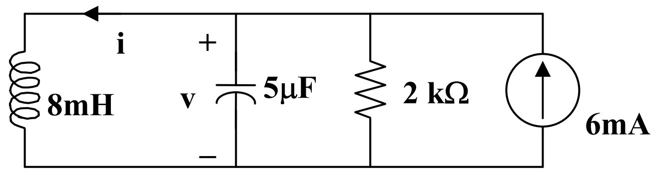
$$di(0)/dt = 0 = 1.323B - 0.5A \text{ or } B = 0.5(-3)/1.323 = -1.134$$

Thus, $$i(t) = \underline{\mathbf{\{4 - [(3\cos 1.323t + 1.134\sin 1.323t)e^{-0.5t}]\}\ A}}$$

Chapter 8, Solution 46.

For $t = 0-$, $u(t) = 0$, so that $v(0) = 0$ and $i(0) = 0$.

For $t > 0$, we have a parallel RLC circuit with a step input, as shown below.



$$\alpha = 1/(2RC) = (1)/(2x2x10^3\ x5x10^{-6}) = 50$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{8x10^3 x5x10^{-6}} = 5{,}000$$

Since $\alpha < \omega_o$, we have an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} \cong -50 \pm j5{,}000$$

Thus, $i(t) = I_s + [(A\cos 5{,}000t + B\sin 5{,}000t)e^{-50t}]$, $I_s = 6mA$

$$i(0) = 0 = 6 + A \text{ or } A = -6mA$$

$$v(0) = 0 = Ldi(0)/dt$$

$$di/dt = [5{,}000(-A\sin 5{,}000t + B\cos 5{,}000t)e^{-50t}] + [-50(A\cos 5{,}000t + B\sin 5{,}000t)e^{-50t}]$$

$$di(0)/dt = 0 = 5{,}000B - 50A \text{ or } B = 0.01(-6) = -0.06mA$$

Thus, $i(t) =$ $\underline{\mathbf{\{6 - [(6\cos 5{,}000t + 0.06\sin 5{,}000t)e^{-50t}]\}\ mA}}$

Chapter 8, Solution 47.

At $t = 0-$, we obtain, $i_L(0) = 3x5/(10 + 5) = 1A$

and $v_o(0) = 0$.

For $t > 0$, the 20-ohm resistor is short-circuited and we have a parallel RLC circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.01) = 10$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.01} = 10$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -10$$

Thus, $$i(t) = I_s + [(A + Bt)e^{-10t}], \quad I_s = 3$$

$$i(0) = 1 = 3 + A \text{ or } A = -2$$

$$v_o = Ldi/dt = [Be^{-10t}] + [-10(A + Bt)e^{-10t}]$$

$$v_o(0) = 0 = B - 10A \text{ or } B = -20$$

Thus, $v_o(t) = $ **$\underline{(200te^{-10t})\ V}$**

Chapter 8, Solution 48.

For $t = 0$-, we obtain $i(0) = -6/(1 + 2) = -2$ and $v(0) = 2x1 = 2$.

For $t > 0$, the voltage is short-circuited and we have a source-free parallel RLC circuit.

$$\alpha = 1/(2RC) = (1)/(2x1x0.25) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.25} = 2$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -2$$

Thus, $$i(t) = [(A + Bt)e^{-2t}], \quad i(0) = -2 = A$$

$$v = Ldi/dt = [Be^{-2t}] + [-2(-2 + Bt)e^{-2t}]$$

$$v_o(0) = 2 = B + 4 \text{ or } B = -2$$

Thus, $i(t) = $ **$\underline{[(-2 - 2t)e^{-2t}]\ A}$**

and $v(t) = $ **$\underline{[(2 + 4t)e^{-2t}]\ V}$**

Chapter 8, Solution 49.

For $t = 0^-$, $i(0) = 3 + 12/4 = 6$ and $v(0) = 0$.

For $t > 0$, we have a parallel *RLC* circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.05) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{5x0.05} = 2$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -2$$

Thus, $i(t) = I_s + [(A + Bt)e^{-2t}]$, $I_s = 3$

$$i(0) = 6 = 3 + A \text{ or } A = 3$$

$$v = Ldi/dt \text{ or } v/L = di/dt = [Be^{-2t}] + [-2(A + Bt)e^{-2t}]$$

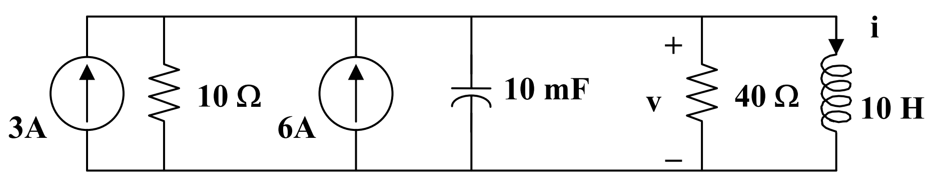
$$v(0)/L = 0 = di(0)/dt = B - 2x3 \text{ or } B = 6$$

Thus, $i(t) =$ **$\{3 + [(3 + 6t)e^{-2t}]\}$ A**

Chapter 8, Solution 50.

For $t = 0-$, $4u(t) = 0$, $v(0) = 0$, and $i(0) = 30/10 = 3A$.

For $t > 0$, we have a parallel RLC circuit.



$$I_s = 3 + 6 = 9A \text{ and } R = 10||40 = 8 \text{ ohms}$$

$$\alpha = 1/(2RC) = (1)/(2x8x0.01) = 25/4 = 6.25$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{4x0.01} = 5$$

Since $\alpha > \omega_o$, we have a overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -10, -2.5$$

Thus, $i(t) = I_s + [Ae^{-10t}] + [Be^{-2.5t}]$, $I_s = 9$

$$i(0) = 3 = 9 + A + B \text{ or } A + B = -6$$

$$di/dt = [-10Ae^{-10t}] + [-2.5Be^{-2.5t}],$$

$$v(0) = 0 = Ldi(0)/dt \text{ or } di(0)/dt = 0 = -10A - 2.5B \text{ or } B = -4A$$

Thus, $A = 2$ and $B = -8$

Clearly, $i(t) =$ **$\{9 + [2e^{-10t}] + [-8e^{-2.5t}]\}$ A**

Chapter 8, Solution 51.

Let i = inductor current and v = capacitor voltage.
At t = 0, v(0) = 0 and $i(0) = i_o$.

For $t > 0$, we have a parallel, source-free LC circuit ($R = \infty$).

$$\alpha = 1/(2RC) = 0 \text{ and } \omega_o = 1/\sqrt{LC} \text{ which leads to } s_{1,2} = \pm j\omega_o$$

$$v = A\cos\omega_o t + B\sin\omega_o t,\ v(0) = 0\ A$$

$$i_C = Cdv/dt = -i$$

$$dv/dt = \omega_o B\sin\omega_o t = -i/C$$

$$dv(0)/dt = \omega_o B = -i_o/C \text{ therefore } B = i_o/(\omega_o C)$$

$v(t) =$ **$-(i_o/(\omega_o C))\sin\omega_o t$ V where $\omega_o = \sqrt{LC}$**

Chapter 8, Solution 52.

$$\alpha = 300 = \frac{1}{2RC} \qquad (1)$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 400 \longrightarrow \omega_o = \sqrt{400^2 - 300^2} = 264.575 = \frac{1}{\sqrt{LC}} \qquad (2)$$

From (2),

$$C = \frac{1}{(264.575)^2 x 50x10^{-3}} = \underline{285.71\mu\text{F}}$$

From (1),

$$R = \frac{1}{2\alpha C} = \frac{1}{2x300}(3500) = \underline{5.833\Omega}$$