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TRABALHO DE TFTD

1) a) $x[n] = (0,3)^n u[n-1]$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n}$$

$$X(\Omega) = \sum_{n=1}^{\infty} (0,3)^n e^{-j\Omega n} = \sum_{n=1} (0,3 e^{-j\Omega})^n = \frac{0,3 e^{-j\Omega}}{1 - 0,3 e^{-j\Omega}}$$

b) $x[n] = (3)^n u[-(n+1)]$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} (3)^n u[-(n+1)]\, e^{-j\Omega n} = \sum_{n=-1}^{-\infty} (3 e^{-j\Omega})^n = \sum_{n=-1} \left(\frac{1}{3} e^{j\Omega}\right)^{-n}$$

Com $m = -n$:

$$X(\Omega) = \sum_{m=1}^{\infty} \left(\frac{1}{3} e^{j\Omega}\right)^m = \frac{\frac{e^{j\Omega}}{3}}{1 - \frac{e^{j\Omega}}{3}}$$

Multiplicando o numerador e o denominador por $\frac{3}{e^{j\Omega}}$:

$$X(\Omega) = \frac{\frac{e^{j\Omega}}{3} \cdot \frac{3}{e^{j\Omega}}}{1 - \frac{e^{j\Omega}}{3} \cdot \frac{3}{e^{j\Omega}}} = \frac{1}{\frac{3}{e^{j\Omega}} - 1} = \frac{1}{3e^{-j\Omega} - 1}$$

2)

a) $x[n] = n a^n u[n]$

$$X(\Omega) = j \frac{d}{d\Omega} \left\{ \frac{e^{j\Omega}}{e^{j\Omega} - a} \right\} = j \frac{e^{j\Omega}(e^{j\Omega} - a) - e^{j\Omega} e^{j\Omega}}{(e^{j\Omega} - a)^2} = j \frac{e^{j2\Omega} - a e^{j\Omega} - e^{j2\Omega}}{(e^{j\Omega} - a)^2}$$

$$X(\Omega) = j \frac{-a e^{j\Omega}}{(e^{j\Omega} - a)^2} = \frac{a e^{j\Omega}}{(e^{j\Omega} - a)^2}$$

b) $x[n] = (n-m) a^{3n} u[n-m]$

$$x[n] = (n-m) a^{2n+n-m+m} u[n-m]$$

$$x[n] = a^{2n+m} (n-m) a^{n-m} u[n-m]$$

$$X(\Omega) = a^{2n+m} \frac{a e^{j\Omega}}{(e^{j\Omega} - a)^2}$$

③ $y[n] + 0{,}7y[n-1] + 0{,}12y[n-2] = 2x[n]$

a) $H(\Omega) = ?$

$$\mathcal{F}\{y[n] + 0{,}7y[n-1] + 0{,}12y[n-2]\} = \mathcal{F}\{2x[n]\}$$

$$Y(\Omega) + 0{,}7e^{-j\Omega}Y(\Omega) + 0{,}12e^{-j2\Omega}Y(\Omega) = 2X(\Omega)$$

$$(1 + 0{,}7e^{-j\Omega} + 0{,}12e^{-j2\Omega})Y(\Omega) = 2X(\Omega)$$

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{2}{1 + 0{,}7e^{-j\Omega} + 0{,}12e^{-j2\Omega}}$$

b) $y[n]$ para $x[n] = (0{,}3)^n u[n]$

$$X(\Omega) = \frac{e^{j\Omega}}{e^{j\Omega} - 0{,}3}$$

$$H(\Omega) = \frac{2e^{j2\Omega}}{e^{j2\Omega} + 0{,}7e^{j\Omega} + 0{,}12} = \frac{2e^{j2\Omega}}{(e^{j\Omega} + 0{,}3)(e^{j\Omega} + 0{,}4)}$$

$$Y(\Omega) = X(\Omega)H(\Omega) = \frac{e^{j\Omega}}{e^{j\Omega} - 0{,}3} \cdot \frac{2e^{j2\Omega}}{(e^{j\Omega} + 0{,}3)(e^{j\Omega} + 0{,}4)} = \frac{2e^{j3\Omega}}{(e^{j\Omega} - 0{,}3)(e^{j\Omega} + 0{,}3)(e^{j\Omega} + 0{,}4)}$$

$$\frac{Y(\Omega)}{e^{j\Omega}} = \frac{2e^{j2\Omega}}{(e^{j\Omega} - 0{,}3)(e^{j\Omega} + 0{,}3)(e^{j\Omega} + 0{,}4)} = \frac{A}{e^{j\Omega} - 0{,}3} + \frac{B}{e^{j\Omega} + 0{,}3} + \frac{C}{e^{j\Omega} + 0{,}4}$$

$$A = \frac{2(0{,}3)^2}{(0{,}3 + 0{,}3)(0{,}3 + 0{,}4)} = \frac{3}{7} \quad B = \frac{2(-0{,}3)^2}{(-0{,}3 - 0{,}3)(-0{,}3 + 0{,}4)} = -3 \quad C = \frac{2(-0{,}4)^2}{(-0{,}4 - 0{,}3)(-0{,}4 + 0{,}3)} = \frac{32}{7}$$

$$\frac{Y(\Omega)}{e^{j\Omega}} = \frac{\frac{3}{7}}{e^{j\Omega} - 0{,}3} - \frac{3}{e^{j\Omega} + 0{,}3} + \frac{\frac{32}{7}}{e^{j\Omega} + 0{,}4}$$

$$Y(\Omega) = \frac{3}{7}\left(\frac{e^{j\Omega}}{e^{j\Omega} - 0{,}3}\right) - 3\frac{e^{j\Omega}}{e^{j\Omega} + 0{,}3} + \frac{32}{7}\left(\frac{e^{j\Omega}}{e^{j\Omega} + 0{,}4}\right)$$

$$y[n] = \frac{3}{7}(0{,}3)^n u[n] - 3(-0{,}3)^n u[n] + \frac{32}{7}(-0{,}4)^n u[n]$$

④ $h[n] = \delta[n] + \delta[n-3]$

$x[n] = 3 + 3\cos\left(\frac{\pi n}{10}\right)$

$H(\Omega) = 1 + e^{-j3\Omega}$

$Y(\Omega) = X(\Omega)H(\Omega)$

$Y(\Omega) = X(\Omega)(1 + e^{-j3\Omega}) = X(\Omega) + X(\Omega)e^{-j3\Omega}$

$y[n] = 3 + 3\cos\left(\frac{\pi n}{10}\right) + 3 + 3\cos\left(\frac{\pi(n-3)}{10}\right)$

$y[n] = 6 + 3\cos\left(\frac{\pi n}{10}\right) + 3\cos\left(\frac{\pi(n-3)}{10}\right)$

⑤ $h[n] = a^n u[n]$. $\rightarrow H(\Omega) = \dfrac{e^{j\Omega}}{e^{j\Omega} - a}$

$x[n] = 3$ $\rightarrow X(\Omega) = 3.\left[2\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)\right]$

$X(\Omega) = 6\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)$

$Y(\Omega) = H(\Omega)X(\Omega)$

$Y(\Omega) = \dfrac{e^{j\Omega}}{e^{j\Omega} - a} \cdot 6\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)$

$Y(\Omega) = \dfrac{e^{j2\pi k}}{e^{j2\pi k} - a} \cdot 6\pi \cdot 1$

$Y(\Omega) = \dfrac{6\pi e^{j2\pi k}}{e^{j2\pi k} - a}$

⑥ $(-1)^n = \cos(\pi n)$ $H_{lp}(\Omega) = \begin{cases} 1 & |\Omega| \leq \frac{\pi}{4} \\ 0 & \frac{\pi}{4} < |\Omega| \leq \pi \end{cases}$

$w_1[n] = x[n]\cos(\pi n)$

$$x[n]\cos(\pi n) \Longleftrightarrow \sum_{m=-\infty}^{\infty} \text{ret}\left(\frac{\Omega - \pi - 2\pi m}{\pi}\right) + \text{ret}\left(\frac{\Omega + \pi - 2\pi m}{\pi}\right) = W_1[\Omega]$$

Sendo a resposta ao impulso do filtro H_{lp}:

$$h_{LP}[n-n_0] = \frac{w_c}{\pi}\,\text{sinc}(w_c(n-n_0))$$
$$= \frac{\frac{\pi}{4}}{\pi}\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right)$$
$$= \frac{1}{4}\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right) \Longleftrightarrow \frac{4}{\pi}\sum_{k=-\infty}^{\infty} \text{ret}\left(\frac{\Omega - 2\pi k}{\frac{\pi}{2}}\right)$$

$w_2[n] = w_1[n] \cdot h_{LP}[n]$

$w_2[n] = w_1[n] \cdot \frac{1}{4}\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right)$

$w_3[n] = w_2[n]\cos(\pi n)$

$w_4[n] = x[n]\,h_{LP}[n]$

$w_4[n] = x[n] \cdot \frac{1}{4}\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right)$

$y[n] = w_3[n] + w_4[n]$

$y[n] = x[n]\cos(\pi n) \cdot \frac{1}{4}\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right) \cdot \cos(\pi n) + x[n] \cdot \frac{1}{4}\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right)$

$y[n] = x[n]\cos^2(\pi n) \cdot \frac{1}{4}\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right) + x[n] \cdot \frac{1}{4}\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right)$

$y[n] = x[n] \cdot \frac{1+\cos(2\pi n)}{2} \cdot \frac{1}{4}\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right) + x[n] \cdot \frac{1}{4}\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right)$

$y[n] = x[n] \cdot 1 \cdot \frac{1}{4}\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right) + x[n] \cdot \frac{1}{4}\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right)$

$y[n] = x[n]\,\frac{1}{4}\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right) + x[n] \cdot \frac{1}{4}\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right)$

$y[n] = 2x[n]\,\frac{1}{4}\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right) = \frac{1}{2}x[n]\,\text{sinc}\left(\frac{\pi}{4}(n-n_0)\right)$

$$Y(\Omega) = \frac{1}{2}X(\Omega)\sum_{k=-\infty}^{\infty} \text{ret}\left(\frac{\Omega - \Omega_0}{\frac{\pi}{2}}\right)$$

O tipo de filtro é um passa-baixa.