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1) Dado $y[0]=1$ e $y[1]=2$ como condições auxiliares e entrada $x[n]=u[n]$,

Resolva: $y[n]+3y[n-1]+2y[n-2]=x[n-1]+3x[n-2]$

Como as condições auxiliares não correspondem ao deslocamento, é preciso deslocar o sinal em 2 de avanço. Portanto:

$y[n+2]+3y[n+1]+2y[n]=x[n+1]+3x[n]$

$z^2Y[z]-z^2y[0]-zy[1]+3\left[zY[z]-zy[0]\right]+2Y[z]=zX[z]-zx[0]+3X[z]$

$z^2Y[z]-z^2(1)-z(2)+3\left[zY[z]-z(1)\right]+2Y[z]=zX[z]-z(1)+3X[z]$

$z^2Y[z]-z^2-2z+3zY[z]-3z+2Y[z]=zX[z]-z+3X[z]$

$Y[z]\left(z^2+3z+2\right)-z^2-2z-3z=X[z]\left(z+3\right)-z$

$Y[z](z^2+3z+2)-z^2-5z=X[z](z+3)-z$

$Y[z](z^2+3z+2)=X[z](z+3)-z+z^2+5z$

$Y[z](z^2+3z+2)=X[z](z+3)+4z+z^2$

$Y[z](z^2+3z+2)=\dfrac{z}{z-1}(z+3)+4z+z^2$

$Y[z](z^2+3z+2)=\dfrac{z(z+3)+4z^2-4z+z^3-z^2}{z-1}$

$Y[z]=\dfrac{z^2+3z+4z^2-4z+z^3-\cancel{z^2}}{(z-1)(z+1)(z+2)}$

$Y[z]=\dfrac{z^3+4z^2-z}{(z-1)(z+1)(z+2)}$

$Y[z]=\dfrac{z(z^2+4z-1)}{(z-1)(z+1)(z+2)}$

$$\frac{Y[z]}{z} = \frac{z^2+4z-1}{(z-1)(z+1)(z+2)}$$

$$A = \frac{1^2+4.1-1}{(1+1)(1+2)} = \frac{1+4-1}{2\cdot 3} = \frac{4}{6} = \frac{2}{3}$$

$$B = \frac{(-1)^2+4(-1)-1}{(-1-1)(-1+2)} = \frac{1-4-1}{-2\cdot 1} = \frac{-4}{-2} = 2$$

$$C = \frac{(-2)^2+4(-2)-1}{(-2-1)(-2+1)} = \frac{4-8-1}{-3\cdot -1} = \frac{-5}{3}$$

$$\frac{Y[z]}{z} = \frac{A}{(z-1)} + \frac{B}{z+1} + \frac{C}{z+2}$$

$$\frac{Y[z]}{z} = \frac{\frac{2}{3}}{z-1} + \frac{2}{z+1} - \frac{\frac{5}{3}}{z+2}$$

$$Y[z] = \frac{2}{3}\left(\frac{z}{z-1}\right) + 2\left(\frac{z}{z+1}\right) - \frac{5}{3}\left(\frac{z}{z+2}\right)$$

$$y[n] = \left(\frac{2}{3}(1)^n + 2(-1)^n - \frac{5}{3}(-2)^n\right)u[n]$$

$$Y[n] = \left(\frac{2}{3} + 2(-1)^n - \frac{5}{3}(-2)^n\right)u[n]$$

Dado $y[-1]=2$ e $y[-2]=0$ como condições iniciais e entrada $x[n]=u[n]$, separe em componentes de entrada nula e estado nulo a solução da equação diferença abaixo!

$$y[n+2] - \frac{5}{6}y[n+1] + \frac{1}{6}y[n] = 5x[n+1] - x[n]$$

a) Entrada nula: deslocar o sistema -2

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = 0$$

$$Y[z] - \frac{5}{6}\left(\frac{1}{z}Y[z] + y[-1]\right) + \frac{1}{6}\left(\frac{1}{z^2}Y[z] + \frac{1}{z}y[-1] + y[-2]\right) = 0$$

$$Y[z] - \frac{5}{6}\left(\frac{1}{z}Y[z] + 2\right) + \frac{1}{6}\left(\frac{1}{z^2}Y[z] + \frac{1}{z}2 + 0\right) = 0$$

$$Y[z] - \frac{5}{6z}Y[z] - \frac{10}{6} + \frac{1}{6z^2}Y[z] + \frac{2}{6z} = 0$$

Multiplicando tudo por z^2:

$$z^2Y[z] - \frac{5z}{6}Y[z] - \frac{5z^2}{3} + \frac{1}{6}Y[z] + \frac{z}{3} = 0$$

$$Y[z]\left(z^2 - \frac{5z}{6} + \frac{1}{6}\right) - \frac{5z^2}{3} + \frac{z}{3} = 0$$

$$Y[z]\left(z^2 - \frac{5z}{6} + \frac{1}{6}\right) = \frac{5z^2}{3} - \frac{z}{3}$$

$$Y[z]\left(\frac{6z^2 - 5z + 1}{6}\right) = \frac{5z^2}{3} - \frac{z}{3}$$

Multiplicando tudo por 6:

$$Y[z](6z^2 - 5z + 1) = 10z^2 - 2z$$

$$Y[z] = \frac{10z^2 - 2z}{(z-\frac{1}{3})(z-\frac{1}{2})}$$

$$Y[z] = \frac{z(10z-2)}{(z-\frac{1}{3})(z-\frac{1}{2})}$$

$$\frac{Y[z]}{z} = \frac{10z-2}{(z-\frac{1}{3})(z-\frac{1}{2})} = \frac{A}{z-\frac{1}{3}} + \frac{B}{z-\frac{1}{2}}$$

$$A = \frac{10\cdot\frac{1}{3} - 2}{(\frac{1}{3}-\frac{1}{2})} = \frac{\frac{10}{3} - 2}{-\frac{1}{6}} = -8$$

$$B = \frac{10\cdot\frac{1}{2} - 2}{(\frac{1}{2}-\frac{1}{3})} = \frac{5-2}{\frac{1}{6}} = \frac{3}{\frac{1}{6}} = 18$$

$$\frac{Y[z]}{z} = -\frac{8}{z-\frac{1}{3}} + \frac{18}{z-\frac{1}{2}}$$

$$Y[z] = -8\frac{z}{z-\frac{1}{3}} + 18\frac{z}{z-\frac{1}{2}}$$

Dividindo tudo por 6:

$$Y[z] = -\frac{4}{3}\left(\frac{z}{z-\frac{1}{3}}\right) + 3\frac{z}{z-\frac{1}{2}}$$

$$y[n] = 3\left(\frac{1}{2}\right)^n u[n] - \frac{4}{3}\left(\frac{1}{3}\right)^n u[n]$$

$$y[n] = \left[3\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{3}\right)^n\right]u[n]$$

b) Estado nulo

$$z^2Y[z] - \frac{5}{6}zY[z] + \frac{1}{6}Y[z] = 5zX[z] - X[z]$$

$$Y[z]\left(z^2 - \frac{5}{6}z + \frac{1}{6}\right) = X[z](5z-1)$$

$$Y[z]\left(z^2 - \frac{5}{6}z + \frac{1}{6}\right) = \frac{z}{z-1}(5z-1)$$

$$Y[z]\left(\frac{6z^2-5z+1}{6}\right)=\frac{z(5z-1)}{z-1}$$

Multiplicando tudo por 6:

$$Y[z](6z^2-5z+1)=\frac{z(30z-6)}{z-1}$$

$$Y[z]=\frac{z(30z-6)}{(z-1)(z-\frac{1}{3})(z-\frac{1}{2})}$$

$$\frac{Y[z]}{z}=\frac{30z-6}{(z-1)(z-\frac{1}{3})(z-\frac{1}{2})}=\frac{A}{z-1}+\frac{B}{z-\frac{1}{3}}+\frac{C}{z-\frac{1}{2}}$$

$$A=\frac{30\cdot 1-6}{(1-\frac{1}{3})(1-\frac{1}{2})}=\frac{24}{\frac{2}{3}\cdot\frac{1}{2}}=72$$

$$B=\frac{30\cdot(\frac{1}{3})-6}{(\frac{1}{3}-1)(\frac{1}{3}-\frac{1}{2})}=\frac{4}{-\frac{2}{3}\cdot(-\frac{1}{6})}=36$$

$$C=\frac{30\cdot\frac{1}{2}-6}{(\frac{1}{2}-1)(\frac{1}{2}-\frac{1}{3})}=\frac{9}{-\frac{1}{2}\cdot\frac{1}{6}}=-108$$

$$\frac{Y[z]}{z}=\frac{72}{z-1}+\frac{36}{z-\frac{1}{3}}-\frac{108}{z-\frac{1}{2}}$$

$$Y[z]=72\frac{z}{z-1}+36\frac{z}{z-\frac{1}{3}}-108\frac{z}{z-\frac{1}{2}}$$

$$y[n]=72\underbrace{(1)^n}_{1}u[n]+36\left(\frac{1}{3}\right)^n u[n]-108\left(\frac{1}{2}\right)^n u[n]$$

$$y[n]=72u[n]+36\left(\frac{1}{3}\right)^n u[n]-108\left(\frac{1}{2}\right)^n u[n]$$

Dividindo tudo por 6:

$$y[n]=12u[n]+6\left(\frac{1}{3}\right)^n u[n]-18\left(\frac{1}{2}\right)^n u[n]$$

$$y[n]=\left[12-18\left(\frac{1}{2}\right)^n+6\left(\frac{1}{3}\right)^n\right]u[n]$$

Dada a equação: $y[n+1]-0{,}5y[n]=x[n]$

determine a resposta em amplitude e fase. Determine a resposta do sistema à entrada senoidal $\cos(1000t-\frac{\pi}{3})$ amostrada a cada $T=0{,}5\,ms$

- Obtendo a função de transferência:

$$zY[z]-0{,}5Y[z]=X[z]$$

$$\frac{Y[z]}{X[z]} = \frac{1}{z-0,5}$$

Assumindo $z = e^{j\Omega}$

$$H(\Omega) = \frac{1}{e^{j\Omega} - 0,5}$$

$$H(\Omega) = \frac{1}{\cos\Omega + j\,\text{sen}\,\Omega - 0,5}$$

$$H(\Omega) = \frac{1}{\cos(\Omega) - 0,5 + j\,\text{sen}(\Omega)}$$

• Resposta em amplitude:

$$|H(\Omega)| = \frac{1}{\sqrt{(\cos(\Omega) - 0,5)^2 + (\text{sen}(\Omega))^2}}$$

$$|H(\Omega)| = \frac{1}{\sqrt{\cos^2(\Omega) - 0,5\cos(\Omega) - 0,5\cos(\Omega) + 0,25 + 0}}$$

$$|H(\Omega)| = \frac{1}{\sqrt{1 - \cos(\Omega) + 0,25}}$$

$$|H(\Omega)| = \frac{1}{\sqrt{1,25 - \cos(\Omega)}} \longrightarrow |H(\Omega)|^2 = \frac{1}{1,25 - \cos(\Omega)}$$

• Fase

$$\angle H(\Omega) = -\tan^{-1}\left[\frac{\text{sen}(\Omega)}{\cos(\Omega) - 0,5}\right]$$

• Resposta a entrada $\cos\left(1000t - \frac{\pi}{3}\right)$, amostrada a cada $T = 0,5\,ms$

Assumindo $t = nT$

$$x[n] = \cos\left(1000nT - \frac{\pi}{3}\right) \qquad x[n] = \cos\left(0,5n - \frac{\pi}{3}\right)$$

neste caso; $\Omega = 0,5$

$$|H(e^{j0,5})| = \frac{1}{\sqrt{1,25 - \cos(0,5)}} = 1,6386$$

$$\angle H(e^{j0,5}) = -\tan^{-1}\left[\frac{\operatorname{sen}(0,5)}{1,25 - \cos(0,5)}\right] = -0,9104$$

$$y[n] = 1,6386 \cos\left(0,5n - \frac{\pi}{3} - 0,9104\right)$$

$$y[n] = 1,6386 \cos(0,5n - 1,9576)$$