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1ª Avaliação de Sinais e Sistemas 2

$\boxed{\text{Questões 1, 2 e 4}}$

1) $y[n] = x[n] - 5y[n-1] - 6y[n-2]$

Com $y[-1] = 2$, $y[-2] = 1$, $x[n] = \delta[n]$

$$Y[z] = X[z] - 5\left[\frac{1}{z}Y[z] + y[-1]\right] - 6\left[\frac{1}{z^2}Y[z] + \frac{1}{z}y[-1] + y[-2]\right]$$

$$Y[z] = 1 - \frac{5}{z}Y[z] - 5y[-1] - \frac{6}{z^2}Y[z] - \frac{6}{z}y[-1] - 6y[-2]$$

$$Y[z] = 1 - \frac{5}{z}Y[z] - 5\cdot 2 - \frac{6}{z^2}Y[z] - \frac{6}{z}\cdot 2 - 6\cdot 1$$

$$Y[z] = 1 - \frac{5}{z}Y[z] - 10 - \frac{6}{z^2}Y[z] - \frac{12}{z} - 6$$

$$Y[z] = -15 - \frac{5}{z}Y[z] - \frac{6}{z^2}Y[z] - \frac{12}{z}$$

$$Y[z] + \frac{5}{z}Y[z] + \frac{6}{z^2}Y[z] = -15 - \frac{12}{z}$$

$$Y[z]\left(1 + \frac{5}{z} + \frac{6}{z^2}\right) = -15 - \frac{12}{z}$$

multiplicando tudo por z^2:

$$Y[z]\left(z^2 + 5z + 6\right) = -15z^2 - 12z$$

$$Y[z] = \frac{-15z^2 - 12z}{(z+2)(z+3)}$$

$$Y[z] = \frac{z(-15z - 12)}{(z+2)(z+3)}$$

$$\frac{Y[z]}{z} = \frac{-15z - 12}{(z+2)(z+3)} = \frac{A}{(z+2)} + \frac{B}{(z+3)}$$

$$A = \frac{-15\cdot(-2) - 12}{-2+3} = \frac{30-12}{1} = \frac{18}{1} = 18$$

$$B = \frac{-15\cdot(-3) - 12}{-3+2} = \frac{45-12}{-1} = \frac{33}{-1} = -33$$

$$\frac{Y[z]}{z} = \frac{18}{z+2} - \frac{33}{z+3}$$

$$Y[z] = 18\left(\frac{z}{z+2}\right) - 33\left(\frac{z}{z+3}\right)$$

$$y[n] = \left[18(-2)^n - 33(-3)^n\right] u[n]$$

2) Sendo $X[z] = \underbrace{\frac{z}{z-\frac{1}{2}}}_{A} + \underbrace{\frac{z}{z-\frac{1}{3}}}_{B} + \underbrace{\frac{z}{z-\frac{1}{4}}}_{C}$

Região de convergência: A) $|z| > \frac{1}{2}$, B) $|z| > \frac{1}{3}$, C) $|z| > \frac{1}{4}$

(i) $|z| \leq \frac{1}{4}$

$$x[n] = -\left(\frac{1}{2}\right)^n u[-n-1] - \left(\frac{1}{3}\right)^n u[-n-1] - \left(\frac{1}{4}\right)^n u[-n-1]$$

$$x[n] = -\left[\left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n + \left(\frac{1}{4}\right)^n\right] u[-n-1]$$

(ii) $|z| \geq \frac{1}{2}$

$$x[n] = -\left(\frac{1}{2}\right)^n u[-n-1] + \left[\left(\frac{1}{3}\right)^n + \left(\frac{1}{4}\right)^n\right] u[n]$$

(iii) $\frac{1}{4} \leq |z| \leq \frac{1}{3}$

$$x[n] = -\left[\left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n + \left(\frac{1}{4}\right)^n\right] u[-n-1].$$

(iv) $\frac{1}{3} \leq |z| \leq \frac{1}{2}$

$$x[n] = -\left[\left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n\right] u[-n-1] + \left(\frac{1}{4}\right)^n u[n]$$

4) Equação: $y[n+1] - 0,8y[n] = x[n+1]$

a) $H[z] = ?$

$$y[n+1] - 0,8y[n] = x[n+1]$$

$$zY[z] - 0,8Y[z] = zX[z]$$

$$Y[z](z - 0,8) = zX[z]$$

$$H[z] = \frac{Y[z]}{X[z]}$$

$$H[z] = \frac{z}{z - 0,8}$$

• Resposta em frequência $H[e^{j\Omega}]$:

Assumindo $z = e^{j\Omega}$

$$H[e^{j\Omega}] = \frac{e^{j\Omega}}{e^{j\Omega} - 0,8}$$

multiplicando numerador e denominador por $e^{-j\Omega}$:

$$H[e^{j\Omega}] = \frac{1}{1 - 0,8e^{-j\Omega}}$$

$$H[e^{j\Omega}] = \frac{1}{1 - 0,8(\cos(\Omega) - j\,\text{sen}(\Omega))}$$

$$H[e^{j\Omega}] = \frac{1}{(1 - 0,8\cos(\Omega)) + 0,8j\,\text{sen}(\Omega)}$$

• Módulo:

$$|H[e^{j\Omega}]| = \frac{1}{\sqrt{(1 - 0,8\cos(\Omega))^2 + (0,8\,\text{sen}(\Omega))^2}}$$

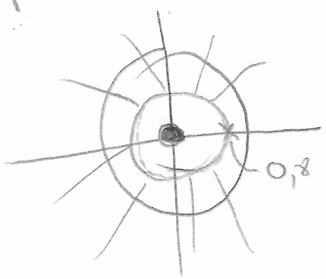
$$|H[e^{j\Omega}]| = \frac{1}{\sqrt{1 - 0{,}8\cos(\Omega) - 0{,}8\cos(\Omega) + 0{,}64\cos^2(\Omega) + 0}}$$

$$|H[e^{j\Omega}]| = \frac{1}{\sqrt{1 - 1{,}6\cos(\Omega) + 0{,}64}} = \frac{1}{\sqrt{1{,}64 - 1{,}6\cos(\Omega)}}$$

- Fase

$$\angle H[e^{j\Omega}] = -\tan^{-1}\left[\frac{0{,}8\,\text{sen}(\Omega)}{1 - 0{,}8\cos(\Omega)}\right]$$

b) Sendo $H[z] = \dfrac{z}{z - 0{,}8}$, $|z| > 0{,}8$. Desta forma, temos $z = 0$ com zero e pólo em $0{,}8$.



O filtro é um passa-baixa estável, já que o pólo encontra-se no semi-plano direito, no interior do círculo unitário.

c) $x[n] = 0{,}5^n u[n]$

$$y[n+1] - 0{,}8\,y[n] = x[n+1]$$

$$zY[z] - 0{,}8\,Y[z] = zX[z]$$

$$zY[z] - 0{,}8\,Y[z] = z \cdot \frac{z}{z - 0{,}5}$$

$$Y[z](z - 0{,}8) = \frac{z^2}{z - 0{,}5}$$

$$Y[z] = \frac{z^2}{(z - 0{,}5)(z - 0{,}8)}$$

$$A = \frac{0,5}{0,5-0,8} = \frac{0,5}{-0,3} = -\frac{5}{3}$$

$$\frac{Y[z]}{z} = \frac{z}{(z-0,5)(z-0,8)} = \frac{A}{z-0,5} + \frac{B}{z-0,8}$$

$$B = \frac{0,8}{0,8-0,5} = \frac{0,8}{0,3} = \frac{8}{3}$$

$$\frac{Y[z]}{z} = -\frac{5}{3}\left(\frac{1}{z-0,5}\right) + \frac{8}{3}\left(\frac{1}{z-0,8}\right)$$

$$Y[z] = -\frac{5}{3}\left(\frac{z}{z-0,5}\right) + \frac{8}{3}\left(\frac{z}{z-0,8}\right)$$

$$y[n] = \left[-\frac{5}{3}(0,5)^n + \frac{8}{3}(0,8)^n\right]u[n]$$

d) $x[n] = \cos(1500t)$, amostrado a $T = 0,001$

- Obtendo a função de transferência

Dada a equação: $y[n+1] - 0,8y[n] = x[n+1]$

Obtém-se: $zY[z] - 0,8Y[z] = zX[z]$

$$\frac{Y[z]}{X[z]} = \frac{z}{z-0,8}$$

Assumindo $z = e^{j\Omega}$

$$H(e^{j\Omega}) = \frac{e^{j\Omega}}{e^{j\Omega}-0,8} = \frac{1}{1-0,8e^{-j\Omega}}$$

$$H(e^{j\Omega}) = \frac{1}{1-0,8(\cos(\Omega) - j\,\mathrm{sen}(\Omega))}$$

$$H(e^{j\Omega}) = \frac{1}{(1-0,8\cos(\Omega) + j0,8\,\mathrm{sen}(\Omega)}$$

• módulo

$$|H(e^{j\Omega})| = \frac{1}{\sqrt{(1-0{,}8\cos(\Omega))^2 + (0{,}8\,\text{sen}(\Omega))^2}}$$

$$|H(e^{j\Omega})| = \frac{1}{\sqrt{1 - 0{,}8\cos(\Omega) - 0{,}8\cos(\Omega) + 0{,}64\cos^2(\Omega) + 0}}$$

$$|H(e^{j\Omega})| = \frac{1}{\sqrt{1 - 1{,}6\cos(\Omega) + 0{,}64}}$$

$$|H(e^{j\Omega})| = \frac{1}{\sqrt{1{,}64 - 1{,}6\cos(\Omega)}}$$

• fase

$$\angle H(e^{j\Omega}) = -\tan^{-1}\left[\frac{0{,}8\,\text{sen}(\Omega)}{1-0{,}8\cos(\Omega)}\right]$$

• Resposta à entrada $\cos(1500t)$, amostrado a cada $T = 0{,}001\,s$

Assumindo $t = nT$

$$x[n] = \cos(1500t) \rightarrow x[n] = \cos(1500nT) \rightarrow x[n] = \cos(1{,}5n)$$

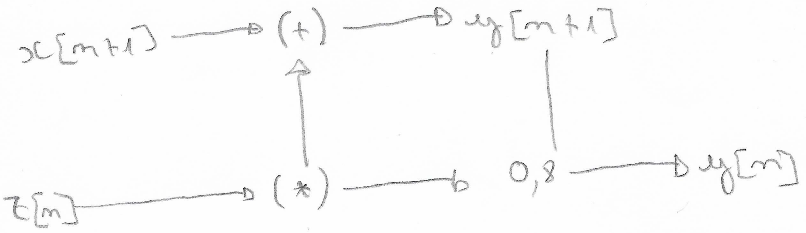
Dessa forma, $\Omega = 1{,}5$;

$$|H(e^{j1{,}5})| = \frac{1}{\sqrt{1{,}64 - 1{,}6\cos(1{,}5)}} = \frac{1}{\sqrt{1{,}64 - 0{,}1132}} = \frac{1}{\sqrt{1{,}5268}} = 0{,}809$$

$$\angle H(e^{j1{,}5}) = -\tan^{-1}\left[\frac{0{,}8\,\text{sen}(1{,}5)}{1-0{,}8\cos(1{,}5)}\right] = -\tan^{-1}\left[\frac{0{,}797995}{0{,}9434}\right] = -0{,}702\,\text{rad}$$

$$y[n] = 0{,}809\cos(1{,}5n - 0{,}702)$$

e) Realização canônica em diagrama de blocos:



f) Para $H[z] = \dfrac{z+1}{z-0,8}$, o zero ficaria no semiplano esquerdo em -1.