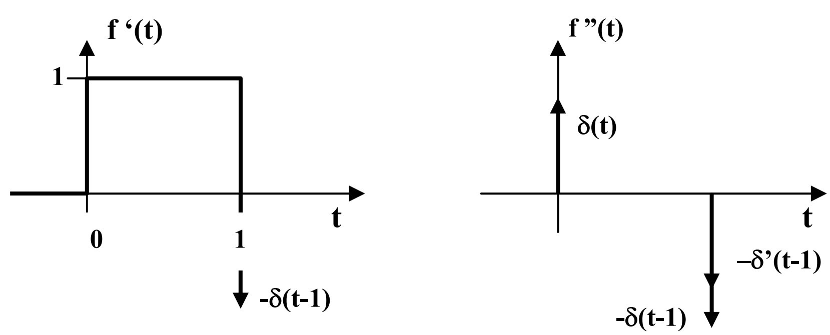
Chapter 18, Solution 1.

$$f'(t) = \delta(t+2) - \delta(t+1) - \delta(t-1) + \delta(t-2)$$
$$j\omega F(\omega) = e^{j2\omega} - e^{j\omega} - e^{-j\omega} + e^{-j\omega 2}$$
$$= 2\cos 2\omega - 2\cos\omega$$

$$F(\omega) = \underline{\mathbf{\frac{2[\cos 2\omega - \cos\omega]}{j\omega}}}$$

Chapter 18, Solution 2.

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$$



f''(t) = δ(t) - δ(t - 1) - δ'(t - 1)

Taking the Fourier transform gives

$$-\omega^2 F(\omega) = 1 - e^{-j\omega} - j\omega e^{-j\omega}$$

$$F(\omega) = \underline{\mathbf{\frac{(1+j\omega)e^{j\omega} - 1}{\omega^2}}}$$

or $F(\omega) = \int_0^1 t\, e^{-j\omega t}\, dt$

But $\int x\, e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1) + c$

$$F(\omega) = \frac{e^{-j\omega}}{(-j\omega)^2}(-j\omega t - 1)\Big|_0^1 = \underline{\mathbf{\frac{1}{\omega^2}\left[(1+j\omega)e^{-j\omega} - 1\right]}}$$

Chapter 18, Solution 3.

$$f(t) = \frac{1}{2}t, \; -2 < t < 2, \qquad f'(t) = \frac{1}{2}, \; -2 < t < 2$$

$$F(\omega) = \int_{-2}^{2} \frac{1}{2} t\, e^{j\omega t} dt = \frac{e^{-j\omega t}}{2(-j\omega)^2}(-j\omega t - 1)\Big|_{-2}^{2}$$

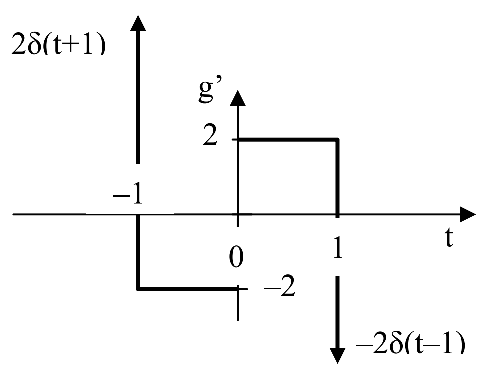
$$= -\frac{1}{2\omega^2}\left[e^{-j\omega 2}(-j\omega 2 - 1) - e^{j\omega 2}(j\omega 2 - 1)\right]$$

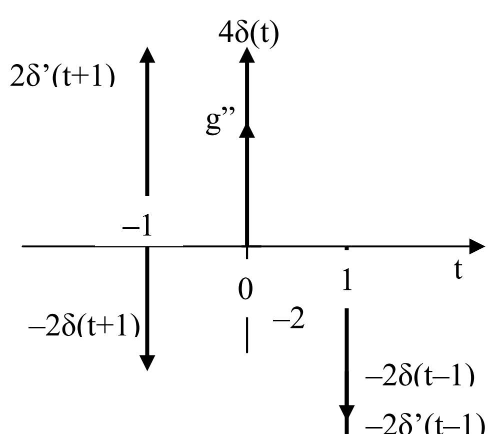
$$= -\frac{1}{2\omega^2}\left[-j\omega 2\left(e^{j\omega 2} + e^{j\omega 2}\right) + e^{j\omega 2} - e^{-j\omega 2}\right]$$

$$= -\frac{1}{2\omega^2}\left(-j\omega 4 \cos 2\omega + j2 \sin 2\omega\right)$$

$$F(\omega) \;=\; \underline{\frac{\mathbf{j}}{\omega^2}(\mathbf{\sin 2\omega - 2\omega \cos 2\omega})}$$

Chapter 18, Solution 4.



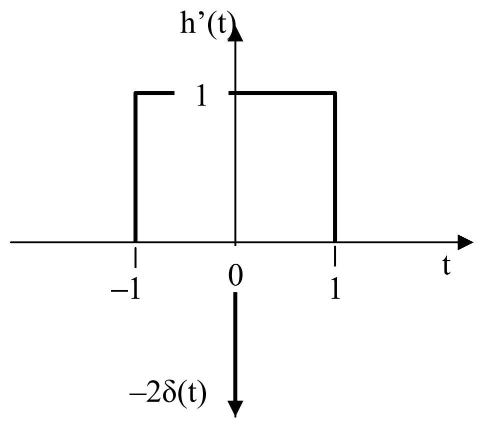


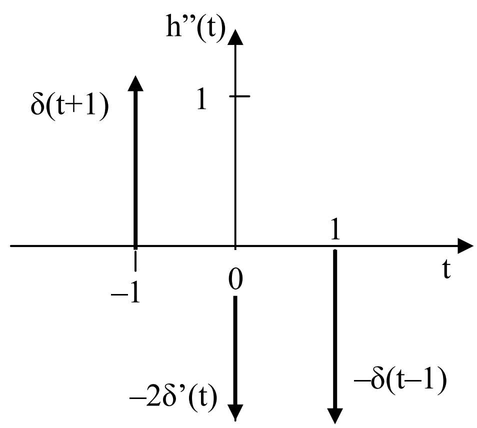
$$g'' = -2\delta(t+1) + 2\delta'(t+1) + 4\delta(t) - 2\delta(t-1) - 2\delta'(t-1)$$

$$(j\omega)^2 G(\omega) = -2e^{j\omega} + 2j\omega e^{j\omega} + 4 - 2e^{-j\omega} - 2j\omega e^{-j\omega}$$
$$= -4\cos\omega - 4\omega\sin\omega + 4$$

$$G(\omega) = \underline{\frac{4}{\omega^2}(\cos\omega + \omega\sin\omega - 1)}$$

Chapter 18, Solution 5.





$$h''(t) = \delta(t+1) - \delta(t-1) - 2\delta'(t)$$

$$(j\omega)^2 H(\omega) = e^{j\omega} - e^{-j\omega} - 2j\omega = 2j\sin\omega - 2j\omega$$

$$H(\omega) = \underline{\frac{2j}{\omega} - \frac{2j}{\omega^2}\sin\omega}$$

Chapter 18, Solution 6.

$$F(\omega) = \int_{-1}^{0} (-1)e^{-j\omega t}dt + \int_{0}^{1} te^{-j\omega t}dt$$

$$\text{Re } F(\omega) = -\int_{-1}^{0} \cos\omega t dt + \int_{0}^{1} t\cos\omega t dt$$

$$= -\frac{1}{\omega}\sin\omega t \Big|_{-1}^{0} + \left(\frac{1}{\omega^2}\cos\omega t + \frac{t}{\omega}\sin\omega t\right)\Big|_{0}^{1} = \underline{\frac{1}{\omega^2}(\cos\omega - 1)}$$

Chapter 18, Solution 7.

(a) f_1 is similar to the function f(t) in Fig. 17.6.

$$f_1(t) = f(t-1)$$

$$\text{Since } F(\omega) = \frac{2(\cos\omega - 1)}{j\omega}$$

$$F_1(\omega) = e^{j\omega}F(\omega) = \underline{\mathbf{\frac{2e^{-j\omega}(\cos\omega - 1)}{j\omega}}}$$

Alternatively,

$$f_1'(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$$

$$j\omega F_1(\omega) = 1 - 2e^{-j\omega} + e^{-j2\omega} = e^{-j\omega}(e^{j\omega} - 2 + e^{j\omega})$$

$$= e^{-j\omega}(2\cos\omega - 2)$$

$$F_1(\omega) = \underline{\mathbf{\frac{2e^{-j\omega}(\cos\omega - 1)}{j\omega}}}$$

(b) f_2 is similar to f(t) in Fig. 17.14.
$f_2(t) = 2f(t)$

$$F_2(\omega) = \underline{\mathbf{\frac{4(1-\cos\omega)}{\omega^2}}}$$

Chapter 18, Solution 8.

(a)
$$F(\omega) = \int_0^1 2e^{-j\omega t}dt + \int_1^2 (4-2t)e^{-j\omega t}dt$$
$$= \frac{2}{-j\omega}e^{-j\omega t}\Big|_0^1 + \frac{4}{-j\omega}e^{-j\omega t}\Big|_1^2 - \frac{2}{-\omega^2}e^{-j\omega t}(-j\omega t-1)\Big|_1^2$$

$$\underline{F(\omega) = \frac{2}{\omega^2} + \frac{2}{j\omega}e^{-j\omega} + \frac{2}{j\omega} - \frac{4}{j\omega}e^{-j2\omega} - \frac{2}{\omega^2}(1+j2\omega)e^{-j2\omega}}$$

(b) g(t) = 2[u(t+2) – u(t-2)] - [u(t+1) – u(t-1)]

$$G(\omega) = \underline{\frac{4\sin 2\omega}{\omega} - \frac{2\sin\omega}{\omega}}$$

Chapter 18, Solution 9.

(a) y(t) = u(t+2) – u(t-2) + 2[u(t+1) – u(t-1)]

$$Y(\omega) = \underline{\frac{2}{\omega}\sin 2\omega + \frac{4}{\omega}\sin\omega}$$

(b) $$Z(\omega) = \int_0^1 (-2t)e^{-j\omega t}dt = \frac{-2e^{-j\omega t}}{-\omega^2}(-j\omega t-1)\Big|_0^1 = \underline{\frac{2}{\omega^2} - \frac{2e^{-j\omega}}{\omega^2}(1+j\omega)}$$

Chapter 18, Solution 10.

(a) $x(t) = e^{2t}u(t)$

$X(\omega) =$ **$\underline{1/(2 + j\omega)}$**

(b)
$$e^{-(t)} = \begin{cases} e^{-t}, & t > 0 \\ e^{t}, & t < 0 \end{cases}$$

$$Y(\omega) = \int_{-1}^{1} y(t)e^{j\omega t}\,dt = \int_{-1}^{0} e^{t}\,e^{j\omega t}\,dt + \int_{0}^{1} e^{-t}\,e^{-j\omega t}\,dt$$

$$= \frac{e^{(1-j\omega)t}}{1-j\omega}\Big|_{-1}^{0} + \frac{e^{-(1+j\omega)t}}{-(1+j\omega)}\Big|_{0}^{1}$$

$$= \frac{2}{1+\omega^2} - e^{-1}\left[\frac{\cos\omega + j\sin\omega}{1-j\omega} + \frac{\cos\omega - j\sin\omega}{1+j\omega}\right]$$

$$Y(\omega) \;=\; \underline{\frac{\mathbf{2}}{\mathbf{1+\omega^2}}\left[\mathbf{1 - e^{-1}(\cos\omega - \omega\sin\omega)}\right]}$$

Chapter 18, Solution 11.

$$f(t) = \sin \pi t\,[u(t) - u(t-2)]$$

$$F(\omega) = \int_0^2 \sin \pi t\, e^{-j\omega t}\, dt = \frac{1}{2j}\int_0^2 \left(e^{j\pi t} - e^{-j\pi t}\right) e^{-j\omega t}\, dt$$

$$= \frac{1}{2j}\left[\int_0^2 (e^{+j(-\omega+\pi)t} + e^{-j(\omega+\pi)t})dt\right]$$

$$= \frac{1}{2j}\left[\frac{1}{-j(\omega-\pi)} e^{-j(\omega-\pi)t}\Big|_0^2 + \frac{e^{-j(\omega+\pi)t}}{-j(\omega+\pi)}\Big|_0^2\right]$$

$$= \frac{1}{2}\left(\frac{1-e^{-j2\omega}}{\pi-\omega} + \frac{1-e^{-j2\omega}}{\pi+\omega}\right)$$

$$= \frac{1}{2(\pi^2-\omega^2)}\left(2\pi + 2\pi e^{-j2\omega}\right)$$

$$F(\omega) \;=\; \underline{\frac{\boldsymbol{\pi}}{\boldsymbol{\omega^2 - \pi^2}}\left(\mathbf{e^{-j\omega 2} - 1}\right)}$$

Chapter 18, Solution 12.

(a) $$F(\omega) = \int_0^\infty e^t\, e^{-j\omega t}\, dt = \int_0^2 e^{(1-j\omega)t}\, dt$$

$$= \frac{1}{1-j\omega} e^{(1-j\omega)t}\Big|_0^2 \;=\; \underline{\frac{\mathbf{e^{2-j\omega 2} - 1}}{\mathbf{1-j\omega}}}$$

(b) $H(\omega) = \int_{-1}^{0} e^{-j\omega t}\, dt + \int_{0}^{1} (-1) e^{-j\omega t}\, dt$

$$= -\frac{1}{j\omega}\left(1 - e^{j\omega}\right) + \frac{1}{j\omega}\left(e^{-j\omega} - 1\right) = \frac{1}{j\omega}(-2 + 2\cos\omega)$$

$$= \frac{-4\sin^2 \omega/2}{j\omega} = \underline{\mathbf{j\omega\left(\frac{\sin\omega/2}{\omega/2}\right)^2}}$$

Chapter 18, Solution 13.

(a) We know that $\mathcal{F}[\cos at] = \pi[\delta(\omega - a) + \delta(\omega + a)]$.

Using the time shifting property,
$\mathcal{F}[\cos a(t - \pi/3a)] = \pi e^{-j\omega\pi/3a}[\delta(\omega - a) + \delta(\omega + a)] = \underline{\pi e^{-j\pi/3}\delta(\omega - a) + \pi e^{j\pi/3}\delta(\omega + a)}$

(b) $\sin\pi(t+1) = \sin\pi t\cos\pi + \cos\pi t\sin\pi = -\sin\pi t$

g(t) = -u(t+1) sin (t+1)

Let $x(t) = u(t)\sin t$, then $X(\omega) = \frac{1}{(j\omega)^2 + 1} = \frac{1}{1 - \omega^2}$

Using the time shifting property,

$$G(\omega) = -\frac{1}{1-\omega^2} e^{j\omega} = \underline{\frac{e^{j\omega}}{\omega^2 - 1}}$$

(c) Let $y(t) = 1 + A\sin at$, then $Y(\omega) = 2\pi\delta(\omega) + j\pi A[\delta(\omega + a) - \delta(\omega - a)]$
$h(t) = y(t)\cos bt$

Using the modulation property,
$$H(\omega) = \frac{1}{2}[Y(\omega + b) + Y(\omega - b)]$$

$$\underline{H(\omega) = \pi\left[\delta(\omega + b) + \delta(\omega - b)\right] + \frac{j\pi A}{2}\left[\delta(\omega + a + b) - \delta(\omega - a + b) + \delta(\omega + a - b) - \delta(\omega - a - b)\right]}$$

(d) $I(\omega) = \int_0^4 (1-t)e^{-j\omega t}dt = \frac{e^{-j\omega t}}{-j\omega} - \frac{e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\Big|_0^4 = \underline{\frac{1}{\omega^2} - \frac{e^{-j4\omega}}{j\omega} - \frac{e^{-j4\omega}}{\omega^2}(j4\omega + 1)}$

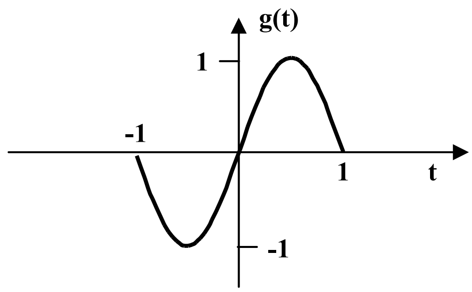
Chapter 18, Solution 14.

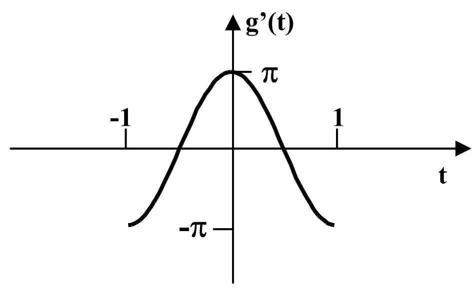
(a) $\cos(3t+\pi) = \cos 3t \cos\pi - \sin 3t \sin\pi = \cos 3t(-1) - \sin 3t(0) = -\cos(3t)$

$f(t) = -e^{-t}\cos 3t\, u(t)$

$$F(\omega) = \underline{\frac{-(1+j\omega)}{(1+j\omega)^2 + 9}}$$

(b)





$g'(t) = \pi\cos\pi t[u(t-1) - u(t-1)]$

$g''(t) = -\pi^2 g(t) - \pi\delta(t+1) + \pi\delta(t-1)$

$-\omega^2 G(\omega) = -\pi^2 G(\omega) - \pi e^{j\omega} + \pi e^{-j\omega}$

$(\pi^2 - \omega^2)G(\omega) = -\pi(e^{j\omega} - e^{-j\omega}) = -2j\pi\sin\omega$

$$G(\omega) = \underline{\frac{2j\pi\sin\omega}{\omega^2 - \pi^2}}$$

Alternatively, we compare this with Prob. 17.7

f(t) = g(t - 1)

$F(\omega) = G(\omega)e^{-j\omega}$

$$G(\omega) = F(\omega)e^{j\omega} = \frac{\pi}{\omega^2 - \pi^2}\left(e^{-j\omega} - e^{j\omega}\right)$$

$$= \frac{-j2\pi\sin\omega}{\omega^2 - \pi^2}$$

$$G(\omega) = \underline{\boldsymbol{\frac{2j\pi\sin\omega}{\pi^2 - \omega^2}}}$$

(c) $\cos\pi(t-1) = \cos\pi t\cos\pi + \sin\pi t\sin\pi = \cos\pi t(-1) + \sin\pi t(0) = -\cos\pi t$

Let $x(t) = e^{-2(t-1)}\cos\pi(t-1)u(t-1) = -e^2 h(t)$

and $y(t) = e^{-2t}\cos(\pi t)u(t)$

$$Y(\omega) = \frac{2 + j\omega}{(2 + j\omega)^2 + \pi^2}$$

$y(t) = x(t-1)$

$Y(\omega) = X(\omega)e^{-j\omega}$

$$X(\omega) = \frac{(2 + j\omega)e^{j\omega}}{(2 + j\omega)^2 + \pi^2}$$

$$X(\omega) = -e^2 H(\omega)$$

$$H(\omega) = -e^{-2}X(\omega)$$

$$= \underline{\boldsymbol{\frac{-(2 + j\omega)e^{j\omega-2}}{(2 + j\omega)^2 + \pi^2}}}$$

(d) Let $x(t) = e^{-2t}\sin(-4t)u(-t) = y(-t)$

$p(t) = -x(t)$

where $y(t) = e^{2t}\sin 4t\,u(t)$

$$Y(\omega) = \frac{2 + j\omega}{(2 + j\omega)^2 + 4^2}$$

$$X(\omega) = Y(-\omega) = \frac{2 - j\omega}{(2 - j\omega)^2 + 16}$$

$$p(\omega) = -X(\omega) = \underline{\frac{\mathbf{j\omega - 2}}{\mathbf{(j\omega - 2)^2 + 16}}}$$

(e) $$Q(\omega) = \frac{8}{j\omega} e^{-j\omega 2} + 3 - 2\left(\pi\delta(\omega) + \frac{1}{j\omega}\right) e^{-j\omega 2}$$

$$Q(\omega) = \underline{\frac{\mathbf{6}}{\mathbf{j\omega}} \mathbf{e^{j\omega 2} + 3 - 2\pi\delta(\omega)e^{-j\omega 2}}}$$

Chapter 18, Solution 15.

(a) $$F(\omega) = e^{j3\omega} - e^{-j\omega 3} = \underline{\mathbf{2j\sin 3\omega}}$$

(b) Let $g(t) = 2\delta(t-1)$, $G(\omega) = 2e^{-j\omega}$

$$F(\omega) = \mathcal{F}\left(\int_{-\infty}^{t} g(t)\,dt\right)$$

$$= \frac{G(\omega)}{j\omega} + \pi F(0)\delta(\omega)$$

$$= \frac{2e^{-j\omega}}{j\omega} + 2\pi\delta(-1)\delta(\omega)$$

$$= \underline{\frac{\mathbf{2e^{-j\omega}}}{\mathbf{j\omega}}}$$

(c) $$\mathcal{F}\left[\delta(2t)\right] = \frac{1}{2} \cdot 1$$

$$F(\omega) = \frac{1}{3} \cdot 1 - \frac{1}{2} j\omega = \underline{\frac{\mathbf{1}}{\mathbf{3}} - \frac{\mathbf{j\omega}}{\mathbf{2}}}$$

Chapter 18, Solution 16.

(a) Using duality properly

$$|t| \;\rightarrow\; \frac{-2}{\omega^2}$$

$$\frac{-2}{t^2} \;\rightarrow\; 2\pi|\omega|$$

or $$\frac{4}{t^2} \;\rightarrow\; -4\pi|\omega|$$

$$F(\omega) \;=\; \mathcal{F}\left(\frac{4}{t^2}\right) = \underline{\mathbf{-4\pi|\omega|}}$$

(b) $$e^{-|a|t} \longrightarrow \frac{2a}{a^2+\omega^2}$$

$$\frac{2a}{a^2+t^2} \longrightarrow 2\pi e^{-a|\omega|}$$

$$\frac{8}{a^2+t^2} \longrightarrow 4\pi e^{-2|\omega|}$$

$$G(\omega) \;=\mathcal{F}\left(\frac{8}{4+t^2}\right) = \underline{\mathbf{4\pi e^{-2|\omega|}}}$$

Chapter 18, Solution 17.

(a) Since $H(\omega) \;=\mathcal{F}\left(\cos\omega_0 t\; f(t)\right) = \frac{1}{2}\left[F(\omega+\omega_0)+F(\omega-\omega_0)\right]$

where $F(\omega) \;=\mathcal{F}\left[u(t)\right] = \pi\delta(\omega) + \frac{1}{j\omega}$, $\omega_0 = 2$

$$H(\omega) = \frac{1}{2}\left[\pi\delta(\omega+2) + \frac{1}{j(\omega+2)} + \pi\delta(\omega-2) + \frac{1}{j(\omega-2)}\right]$$

$$= \frac{\pi}{2}[\delta(\omega+2)+\delta(\omega-2)] - \frac{j}{2}\left[\frac{\omega+2+\omega-2}{(\omega+2)(\omega-2)}\right]$$

$$H(\omega) = \underline{\boldsymbol{\frac{\pi}{2}[\delta(\omega+2)+\delta(\omega-2)] - \frac{j\omega}{\omega^2-4}}}$$

(b) $G(\omega) = \mathcal{F}\,[\sin\omega_0 t\; f(t)] = \frac{j}{2}[F(\omega+\omega_0) - F(\omega-\omega_0)]$

where $F(\omega) = \mathcal{F}\,[u(t)] = \pi\delta(\omega) + \frac{1}{j\omega}$

$$G(\omega) = \frac{j}{2}\left[\pi\delta(\omega+10) + \frac{1}{j(\omega+10)} - \pi\delta(\omega-10) - \frac{1}{j(\omega-10)}\right]$$

$$= \frac{j\pi}{2}[\delta(\omega+10) - \delta(\omega-10)] + \frac{j}{2}\left[\frac{j}{\omega-10} - \frac{j}{\omega+10}\right]$$

$$= \underline{\boldsymbol{\frac{j\pi}{2}[\delta(\omega+10) - \delta(\omega-10)] - \frac{10}{\omega^2-100}}}$$

Chapter 18, Solution 18.

Let $f(t) = e^{-t}u(t) \longrightarrow F(\omega) = \frac{1}{j+j\omega}$

$$f(t)\cos t \longrightarrow \frac{1}{2}[F(\omega-1) + F(\omega+1)]$$

Hence $Y(\omega) = \frac{1}{2}\left[\frac{1}{1+j(\omega-1)} + \frac{1}{1+j(\omega+1)}\right]$

$$= \frac{1}{2}\left[\frac{1+j\omega+j+1+j\omega-j}{[1+j(\omega-1)][1+j(\omega+1)]}\right]$$

$$= \frac{1+j\omega}{1+j\omega+j+j\omega-j-\omega^2+1}$$

$$= \underline{\boldsymbol{\frac{1+j\omega}{2j\omega-\omega^2+2}}}$$

Chapter 18, Solution 19.

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{j\omega t}\,dt = \frac{1}{2}\int_0^1 \left(e^{j2\pi t} + e^{-j2\pi t}\right)e^{-j\omega t}\,dt$$

$$F(\omega) = \frac{1}{2}\int_0^1 \left[e^{-j(\omega+2\pi)t} + e^{-j(\omega-2\pi)t}\right]dt$$

$$= \frac{1}{2}\left[\frac{1}{-j(\omega+2\pi)}e^{-j(\omega+2\pi)t} + \frac{1}{-j(\omega-2\pi)}e^{-j(\omega-2\pi)t}\right]_0^1$$

$$= -\frac{1}{2}\left[\frac{e^{-j(\omega+2\pi)}-1}{j(\omega+2\pi)} + \frac{e^{-j(\omega-2\pi)}-1}{j(\omega-2\pi)}\right]$$

But $\quad e^{j2\pi} = \cos 2\pi + j\sin 2\pi = 1 = e^{-j2\pi}$

$$F(\omega) = -\frac{1}{2}\left(\frac{e^{-j\omega}-1}{j}\right)\left(\frac{1}{\omega+2\pi} + \frac{1}{\omega-2\pi}\right)$$

$$= \underline{\mathbf{\frac{j\omega}{\omega^2 - 4\pi^2}\left(e^{-j\omega}-1\right)}}$$

Chapter 18, Solution 20.

(a) $\quad F(c_n) = c_n\delta(\omega)$

$$F\left(c_n e^{jn\omega_o t}\right) = c_n\delta(\omega - n\omega_o)$$

$$F\left(\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}\right) = \underline{\sum_{\mathbf{n}=-\infty}^{\infty} \mathbf{c_n\delta(\omega - n\omega_o)}}$$

(b) $\quad T = 2\pi \longrightarrow \omega_o = \frac{2\pi}{T} = 1$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}\,dt = \frac{1}{2\pi}\left(\int_0^{\pi} 1\cdot e^{-jnt}\,dt + 0\right)$$

$$= \frac{1}{2\pi}\left(-\frac{1}{jn}e^{jnt}\Big|_0^{\pi}\right) = \frac{j}{2\pi n}\left(e^{-jn\pi} - 1\right)$$

But $e^{-jn\pi} = \cos n\pi + j\sin n\pi = \cos n\pi = (-1)^n$

$$c_n = \frac{j}{2n\pi}\left[(-1)^n - 1\right] = \begin{cases} 0, & n=\text{even} \\ \frac{-j}{n\pi}, & n=\text{odd}, n\neq 0 \end{cases}$$

for n = 0

$$c_n = \frac{1}{2\pi}\int_0^{\pi} 1\,dt = \frac{1}{2}$$

Hence

$$f(t) = \frac{1}{2} - \sum_{\substack{n=-\infty \\ n\neq 0 \\ n=\text{odd}}}^{\infty} \frac{j}{n\pi}e^{jnt}$$

$$F(\omega) = \underline{\boldsymbol{\frac{1}{2}\delta\omega - \sum_{\substack{n=-\infty \\ n\neq 0 \\ n=\text{odd}}}^{\infty} \frac{j}{n\pi}\delta(\omega - n)}}$$

Chapter 18, Solution 21.

Using Parseval's theorem,

$$\int_{-\infty}^{\infty} f^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2\, d\omega$$

If f(t) = u(t+a) – u(t+a), then

$$\int_{-\infty}^{\infty} f^2(t)dt = \int_{-a}^{a} (1)^2 dt = 2a = \frac{1}{2\pi}\int_{-\infty}^{\infty} 4a^2\left(\frac{\sin a\omega}{a\omega}\right)^2 d\omega$$

or

$$\int_{-\infty}^{\infty}\left(\frac{\sin a\omega}{a\omega}\right)^2 d\omega = \frac{4\pi a}{4a^2} = \frac{\pi}{a} \quad \text{as required.}$$

Chapter 18, Solution 22.

$$F\left[f(t)\sin\omega_o t\right] = \int_{-\infty}^{\infty} f(t)\frac{\left(e^{j\omega_o t} - e^{-j\omega_o t}\right)}{2j}e^{-j\omega t}\,dt$$

$$= \frac{1}{2j}\left[\int_{-\infty}^{\infty} f(t)e^{-j(\omega-\omega_o)t}\,dt - \int_{-\infty}^{\infty} e^{-j(\omega+\omega_o)t}\,dt\right]$$

$$= \underline{\frac{\mathbf{1}}{\mathbf{2j}}\left[\mathbf{F}(\omega-\omega_o) - \mathbf{F}(\omega+\omega_o)\right]}$$

Chapter 18, Solution 23.

(a) f(3t) leads to $\frac{1}{3}\cdot\frac{10}{(2+j\omega/3)(5+j\omega/3)} = \frac{30}{(6+j\omega)(15+j\omega)}$

$$F\left[f(-3t)\right] = \underline{\frac{\mathbf{30}}{(\mathbf{6}-\mathbf{j}\omega)(\mathbf{15}-\mathbf{j}\omega)}}$$

(b) $f(2t) \longrightarrow \frac{1}{2}\cdot\frac{10}{(2+j\omega/2)(15+j\omega/2)} = \frac{20}{(4+j\omega)(10+j\omega)}$

$f(2t-1) = f\,[2(t-1/2)] \longrightarrow \underline{\frac{\mathbf{20e^{-j\omega/2}}}{(\mathbf{4}+\mathbf{j}\omega)(\mathbf{10}+\mathbf{j}\omega)}}$

(c) $f(t)\cos 2t \longrightarrow \frac{1}{2}F(\omega+2) + \frac{1}{2}F(\omega+2)$

$$= \underline{\frac{\mathbf{5}}{[\mathbf{2}+\mathbf{j}(\omega+\mathbf{2})][\mathbf{5}+\mathbf{j}(\omega+\mathbf{2})]} + \frac{\mathbf{5}}{[\mathbf{2}+\mathbf{j}(\omega-\mathbf{2})[\mathbf{5}+\mathbf{j}(\omega-\mathbf{2})]]}}$$

(d) $F\left[f'(t)\right] = j\omega F(\omega) = \frac{j\omega 10}{(2+j\omega)(5+j\omega)}$

(e) $\int_{-\infty}^{t} f(t)dt \longrightarrow \frac{F(\omega)}{j(\omega)} + \pi F(0)\delta(\omega)$

$$= \frac{10}{j\omega(2+j\omega)(5+j\omega)} + \pi\delta(\omega)\frac{x10}{2x5}$$

$$= \underline{\mathbf{\frac{10}{j\omega(2+j\omega)(5+j\omega)} + \pi\delta(\omega)}}$$

Chapter 18, Solution 24.

(a) $X(\omega) = F(\omega) + \mathcal{F}[3]$

$$= \underline{\mathbf{6\pi\delta(\omega) + \frac{j}{\omega}\left(e^{-j\omega} - 1\right)}}$$

(b) $y(t) = f(t-2)$

$$Y(\omega) = e^{-j\omega 2}F(\omega) = \underline{\mathbf{\frac{je^{-j2\omega}}{\omega}\left(e^{-j\omega} - 1\right)}}$$

(c) If $h(t) = f'(t)$

$$H(\omega) = j\omega F(\omega) = j\omega\frac{j}{\omega}\left(e^{-j\omega} - 1\right) = \underline{\mathbf{1 - e^{-j\omega}}}$$

(d) $g(t) = 4f\left(\frac{2}{3}t\right) + 10f\left(\frac{5}{3}t\right)$, $G(\omega) = 4x\frac{3}{2}F\left(\frac{3}{2}\omega\right) + 10x\frac{3}{5}F\left(\frac{3}{5}\omega\right)$

$$= 6\cdot\frac{j}{\frac{3}{2}\omega}\left(e^{-j3\omega/2} - 1\right) + \frac{6j}{\frac{3}{5}\omega}\left(e^{-j3\omega/5} - 1\right)$$

$$= \underline{\mathbf{\frac{j4}{\omega}\left(e^{-j3\omega/2} - 1\right) + \frac{j10}{\omega}\left(e^{-j3\omega/5} - 1\right)}}$$

Chapter 18, Solution 25.

(a) $$F(s) = \frac{10}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}, \quad s = j\omega$$

$$A = \frac{10}{2} = 5, \; B = \frac{10}{-2} = -5$$

$$F(\omega) = \frac{5}{j\omega} - \frac{5}{j\omega + 2}$$

$$f(t) = \underline{\mathbf{\frac{5}{2}sgn(t) - 5e^{-2t}u(t)}}$$

(b) $$F(\omega) = \frac{j\omega - 4}{(j\omega+1)(j\omega+2)} = \frac{A}{j\omega+1} + \frac{B}{j\omega+2}$$

$$F(s) = \frac{s-4}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}, \quad s = j\omega$$

$$A = 5, \; B = 6$$

$$F(\omega) = \frac{-5}{1+j\omega} + \frac{6}{2+j\omega}$$

$$f(t) = \underline{\mathbf{(-5e^{-t} + 6e^{-2t})u(t)}}$$

Chapter 18, Solution 26.

(a) $\underline{f(t) = e^{-(t-2)}u(t)}$

(b) $\underline{h(t) = te^{-4t}u(t)}$

(c) If $x(t) = u(t+1) - u(t-1) \longrightarrow X(\omega) = 2\frac{\sin\omega}{\omega}$

By using duality property,

$$G(\omega) = 2u(\omega+1) - 2u(\omega-1) \quad \longrightarrow \quad \underline{g(t) = \frac{2\sin t}{\pi t}}$$

Chapter 18, Solution 27.

(a) Let $F(s) = \frac{100}{s(s+10)} = \frac{A}{s} + \frac{B}{s+10}, \quad s = j\omega$

$$A = \frac{100}{10} = 10, \; B = \frac{100}{-10} = -10$$

$$F(\omega) = \frac{10}{j\omega} - \frac{10}{j\omega+10}$$

$$f(t) = \underline{\mathbf{5sgn(t) - 10e^{-10t}u(t)}}$$

(b) $G(s) = \frac{10s}{(2-s)(3+s)} = \frac{A}{2-s} + \frac{B}{s+3}, \quad s = j\omega$

$$A = \frac{20}{5} = 4, \quad B = \frac{-30}{5} = -6$$

$$G(\omega) = \frac{4}{= -j\omega + 2} - \frac{6}{j\omega+3}$$

$$g(t) = \underline{\mathbf{4e^{2t}u(-t) - 6e^{-3t}u(t)}}$$

(c) $H(\omega) = \frac{60}{(j\omega)^2 + j40\omega + 1300} = \frac{60}{(j\omega+20)^2 + 900}$

$$h(t) = \underline{\mathbf{2e^{-20t}\sin(30t)u(t)}}$$

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\delta(\omega)e^{j\omega t}d\omega}{(2+j\omega)(j\omega+1)} = \frac{1}{2}\pi \cdot \frac{1}{2} = \underline{\mathbf{\frac{1}{4}}}\pi$$

Chapter 18, Solution 28.

(a) $$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}\,d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\pi\delta(\omega)\,e^{j\omega t}}{(5+j\omega)(2+j\omega)}d\omega$$

$$=\frac{1}{2}\,\frac{1}{(5)(2)}=\frac{1}{20}=\underline{\mathbf{0.05}}$$

(b) $$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{10\delta(\omega+2)}{j\omega(j\omega+1)}e^{j\omega t}\,d\omega=\frac{10}{2\pi}\,\frac{e^{-j2t}}{(-j2)(-j2+1)}$$

$$=\frac{j5}{2\pi}\,\frac{e^{-j2t}}{1-j2}=\underline{\frac{\mathbf{(-2+j)e^{-j2t}}}{\mathbf{2\pi}}}$$

(c) $$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{20\delta(\omega-1)e^{j\omega t}}{(2+j\omega)(3+5\omega)}d\omega=\frac{20}{2\pi}\,\frac{e^{jt}}{(2+j)(3+j)}$$

$$=\frac{20e^{jt}}{2\pi(5+5j)}=\underline{\frac{\mathbf{(1-j)e^{jt}}}{\mathbf{\pi}}}$$

(d) Let $$F(\omega)=\frac{5\pi\delta(\omega)}{(5+j\omega)}+\frac{5}{j\omega(5+j\omega)}=F_1(\omega)+F_2(\omega)$$

$$f_1(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{5\pi\delta(\omega)}{5+j\omega}e^{j\omega t}\,d\omega=\frac{5\pi}{2\pi}\cdot\frac{1}{5}=0.5$$

$$F_2(s)=\frac{5}{s(5+s)}=\frac{A}{s}+\frac{B}{s+5}\ ,\ A=1,B=-1$$

$$F_2(\omega)=\frac{1}{j\omega}-\frac{1}{j\omega+5}$$

$$f_2(t)=\frac{1}{2}\operatorname{sgn}(t)-e^{-5t}\ =-\frac{1}{2}+u(t)-e^{5t}$$

$$f(t)=f_1(t)+f_2(t)=\ \underline{\mathbf{u(t)-e^{-5t}}}$$

Chapter 18, Solution 29.

(a) $f(t) = F^{-1}[\delta(\omega)] + F^{-1}\left[4\delta(\omega+3) + 4\delta(\omega-3)\right]$

$$= \frac{1}{2\pi} + \frac{4\cos 3t}{\pi} = \underline{\mathbf{\frac{1}{2\pi}\left(1 + 8\cos 3t\right)}}$$

(b) If $h(t) = u(t+2) - u(t-2)$

$$H(\omega) = \frac{2\sin 2\omega}{\omega}$$

$$G(\omega) = 4H(\omega) \longrightarrow \qquad g(t) = \frac{1}{2\pi}\cdot\frac{8\sin 2t}{t}$$

$$g(t) = \underline{\mathbf{\frac{4\sin 2t}{\pi t}}}$$

(c) Since
$\cos(at) \leftrightarrow \pi\delta(\omega+a) + \pi\delta(\omega-a)$
Using the reversal property,
$2\pi\cos 2\omega \leftrightarrow \pi\delta(t+2) + \pi\delta(t-2)$

or $F^{-1}\left[6\cos 2\omega\right] = \underline{\mathbf{3\delta(t+2) + 3\delta(t-2)}}$

Chapter 18, Solution 30.

(a) $y(t) = \text{sgn}(t) \longrightarrow \quad Y(\omega) = \frac{2}{j\omega}, \quad X(\omega) = \frac{1}{a + j\omega}$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2(a+j\omega)}{j\omega} = 2 + \frac{2a}{j\omega} \longrightarrow \underline{h(t) = 2\delta(t) + a[u(t) - u(-t)]}$$

(b) $X(\omega) = \frac{1}{1+j\omega}, \quad Y(\omega) = \frac{1}{2+j\omega}$

$$H(\omega) = \frac{1+j\omega}{2+j\omega} = 1 - \frac{1}{2+j\omega} \longrightarrow \underline{h(t) = \delta(t) - e^{-2t}u(t)}$$

(c) In this case, by definition, $\underline{h(t) = \quad y(t) = e^{-at}\sin bt\, u(t)}$

Chapter 18, Solution 31.

(a) $Y(\omega) = \dfrac{1}{(a + j\omega)^2}, \qquad H(\omega) = \dfrac{1}{a + j\omega}$

$$X(\omega) = \frac{Y(\omega)}{H(\omega)} = \frac{1}{a + j\omega} \longrightarrow \underline{x(t) = e^{-at}u(t)}$$

(b) By definition, $\underline{x(t) = y(t) = u(t+1) - u(t-1)}$

(c) $Y(\omega) = \dfrac{1}{(a + j\omega)}, \qquad H(\omega) = \dfrac{2}{j\omega}$

$$X(\omega) = \frac{Y(\omega)}{H(\omega)} = \frac{j\omega}{2(a + j\omega)} = \frac{1}{2} - \frac{a}{2(a + j\omega)} \longrightarrow \underline{x(t) = \frac{1}{2}\delta(t) - \frac{a}{2}e^{-at}u(t)}$$

Chapter 18, Solution 32.

(a) Since $\dfrac{e^{-j\omega}}{j\omega + 1}$ $\qquad e^{-(t-1)}u(t-1)$

and $F(-\omega) \longrightarrow f(-t)$

$$F_1(\omega) = \frac{e^{j\omega}}{-j\omega + 1} \longrightarrow f_1(t) = e^{-(-t-1)}u(-t-1)$$

$$f_1(t) = \underline{\mathbf{e^{(t+1)}u(-t-1)}}$$

(b) From Section 17.3,

$$\frac{2}{t^2 + 1} \longrightarrow 2\pi e^{-|\omega|}$$

If $F_2(\omega) = 2e^{-|\omega|}$, then

$$f_2(t) = \underline{\mathbf{\frac{2}{\pi(t^2 + 1)}}}$$

(b) By partial fractions

$$F_3(\omega) = \frac{1}{(j\omega+1)^2(j\omega-1)^2} = \frac{\frac{1}{4}}{(j\omega+1)^2} + \frac{\frac{1}{4}}{(j\omega+1)} + \frac{\frac{1}{4}}{(j\omega-1)^2} - \frac{\frac{1}{4}}{j\omega-1}$$

Hence $f_3(t) = \frac{1}{4}\left(te^{-t} + e^{-t} + te^{t} - e^{t}\right)u(t)$

$$= \underline{\frac{1}{4}(t+1)e^{-t}u(t) + \frac{1}{4}(t-1)e^{t}u(t)}$$

(d) $$f_4(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\omega)e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\delta(\omega)e^{j\omega t}}{1+j2\omega} = \underline{\frac{1}{2\pi}}$$

Chapter 18, Solution 33.

(a) Let $x(t) = 2\sin \pi t[u(t+1) - u(t-1)]$

From Problem 17.9(b),

$$X(\omega) = \frac{4j\pi \sin\omega}{\pi^2 - \omega^2}$$

Applying duality property,

$$f(t) = \frac{1}{2\pi}X(-t) = \frac{2j\sin(-t)}{\pi^2 - t^2}$$

$$f(t) = \underline{\frac{2j\sin t}{t^2 - \pi^2}}$$

(b) $$F(\omega) = \frac{j}{\omega}(\cos 2\omega - j\sin 2\omega) - \frac{j}{\omega}(\cos\omega - j\sin\omega)$$

$$= \frac{j}{\omega}\left(e^{j2\omega} - e^{-j\omega}\right) = \frac{e^{-j\omega}}{j\omega} - \frac{e^{j2\omega}}{j\omega}$$

$$f(t) = \frac{1}{2}\operatorname{sgn}(t-1) - \frac{1}{2}\operatorname{sgn}(t-2)$$

But $\operatorname{sgn}(t) = 2u(t) - 1$

$$f(t) = u(t-1) - \frac{1}{2} - u(t-2) + \frac{1}{2}$$

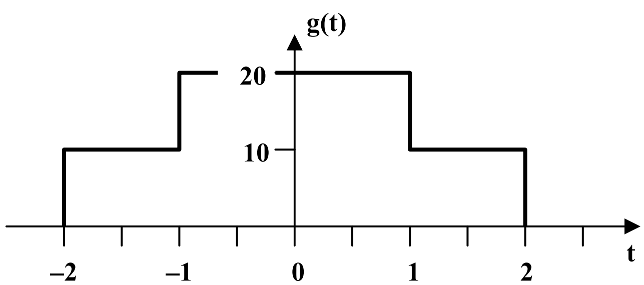
$$= \underline{\mathbf{u(t-1) - u(t-2)}}$$

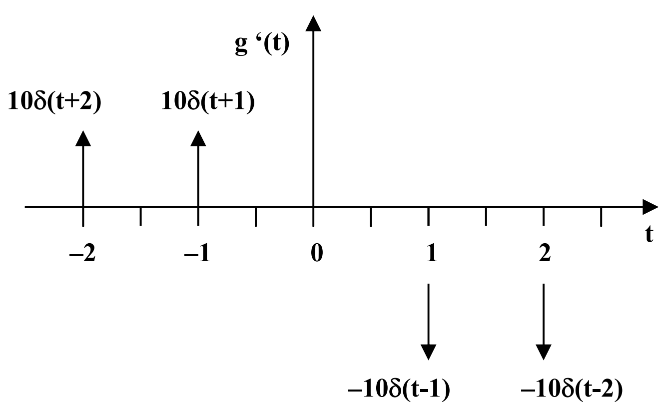
Chapter 18, Solution 34.

First, we find G(ω) for g(t) shown below.

$$g(t) = 10[u(t+2) - u(t-2)] + 10[u(t+1) - u(t-1)]$$
$$g'(t) = 10[\delta(t+2) - \delta(t-2)] + 10[\delta(t+1) - \delta(t-1)]$$

The Fourier transform of each term gives





$$j\omega G(\omega) = 10\left(e^{j\omega 2} - e^{-j\omega 2}\right) + 10\left(e^{j\omega} - e^{-j\omega}\right)$$
$$= 20j\sin 2\omega + 20j\sin\omega$$

$$G(\omega) = \frac{20\sin 2\omega}{\omega} + \frac{20\sin\omega}{\omega} = 40\ \mathrm{sinc}(2\omega) + 20\ \mathrm{sinc}(\omega)$$

Note that G(ω) = G(-ω).

$$F(\omega) = 2\pi G(-\omega)$$

$$f(t) = \frac{1}{2\pi} G(t)$$

$= \underline{\mathbf{(20/\pi)sinc(2t) + (10/\pi)sinc(t)}}$

Chapter 18, Solution 35.

(a) x(t) = f[3(t-1/3)]. Using the scaling and time shifting properties,''

$$X(\omega) = \frac{1}{3}\frac{1}{2 + j\omega/3} e^{-j\omega/3} = \underline{\frac{e^{-j\omega/3}}{(6 + j\omega)}}$$

(b) Using the modulation property,

$$Y(\omega) = \frac{1}{2}[F(\omega+5) + F(\omega-5)] = \frac{1}{2}\left[\frac{1}{2 + j(\omega+5)} + \frac{1}{2 + j(\omega-5)}\right] = \underline{\frac{1}{2}\left[\frac{1}{j\omega+7} + \frac{1}{j\omega-3}\right]}$$

(c) $Z(\omega) = j\omega F(\omega) = \underline{\dfrac{j\omega}{2 + j\omega}}$

(d) $H(\omega) = F(\omega)F(\omega) = \underline{\dfrac{1}{(2 + j\omega)^2}}$

(e) $I(\omega) = j\dfrac{d}{d\omega}F(\omega) = j\dfrac{(0 - j)}{(2 + j\omega)^2} = \underline{\dfrac{1}{(2 + j\omega)^2}}$

Chapter 18, Solution 36.

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$

$$V_o(\omega) = H(\omega)V_i(\omega) = \frac{10V_i(\omega)}{2 + j\omega}$$

(a) $v_i = 4\delta(t) \longrightarrow V_i(\omega) = 4$

$$V_o(\omega) = \frac{40}{2 + j\omega}$$

$$v_o(t) = 40e^{-2t}u(t)$$
$v_o(2) = 40e^{-4} =$ **0.7326 V**

(b) $v_i = 6e^{-t}u(t) \longrightarrow V_i(\omega) = \frac{6}{1 + j\omega}$

$$V_o(\omega) = \frac{60}{(2 + j\omega)(1 + j\omega)}$$

$$V_o(s) = \frac{60}{(s + 2)(s + 1)} = \frac{A}{s + 1} + \frac{B}{s + 2}, \quad s = j\omega$$

$$A = \frac{60}{1} = 60, \quad B = \frac{60}{-1} = -60$$

$$V_o(\omega) = \frac{60}{1 + j\omega} - \frac{60}{2 + j\omega}$$

$$v_o(t) = 60\left[e^{-t} - e^{-2t}\right]u(t)$$
$$v_o(2) = 60\left[e^{-2} - e^{-4}\right] = 60(0.13533 - 0.01831)$$
$=$ **7.021 V**

(c) $v_i(t) = 3 \cos 2t$

$V_i(\omega) = \pi[\delta(\omega + 2) + \delta(\omega- 2)]$

$$V_o = \frac{10\pi[\delta(\omega + 2) + \delta(\omega - 2)]}{2 + j\omega}$$

$$v_o(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} V_o(\omega)e^{j\omega t}d\omega$$

$$= 5\int_{-\infty}^{\infty} \frac{\delta(\omega + 2)}{2 + j\omega}e^{j\omega t}d\omega + 5\int_{-\infty}^{\infty} \frac{\delta(\omega - 2)e^{j\omega t}}{2 + j\omega}d\omega$$

$$= \frac{5e^{-j2t}}{2-j2} + \frac{5e^{+j2t}}{2+j2} = \frac{5}{2\sqrt{2}}\left[e^{-j(2t-45°)} + e^{j(2t-45°)}\right]$$

$$= \frac{5}{\sqrt{2}}\cos(2t-45°)$$

$$v_o(2) = \frac{5}{\sqrt{2}}\cos(4-45°) = \frac{5}{\sqrt{2}}\cos(229.18° - 45°)$$

$v_o(2)$ = **$\underline{-3.526\ V}$**

Chapter 18, Solution 37.

$$2 \| j\omega = \frac{j2\omega}{2+j\omega}$$

By current division,

$$H(\omega) = \frac{I_o(\omega)}{I_s(\omega)} = \frac{\dfrac{j2\omega}{2+j\omega}}{4 + \dfrac{j2\omega}{2+j\omega}} = \frac{j2\omega}{j2\omega + 8 + j4\omega}$$

$$H(\omega) = \underline{\boldsymbol{\frac{j\omega}{4+j3\omega}}}$$

Chapter 18, Solution 38.

$$V_i(\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$V_o(\omega) = \frac{10}{10+j\omega 2}V_i(\omega) = \frac{5}{5+j\omega}\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)$$

$$\text{Let } V_o(\omega) = V_1(\omega) + V_2(\omega) = \frac{5\pi\delta(\omega)}{5+j\omega} + \frac{5}{j\omega(5+j\omega)}$$

$$V_2(\omega) = \frac{5}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5} \longrightarrow A = 1,\ B = -1, \quad s = j\omega$$

$$V_2(\omega) = \frac{1}{j\omega} - \frac{1}{5+j\omega} \longrightarrow v_2(t) = \frac{1}{2}\text{sgn}(t) - e^{-5t}$$

$$V_1 = \frac{5\pi\delta(\omega)}{5+j\omega} \longrightarrow v_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{5\pi\delta(\omega)}{5+j\omega} e^{j\omega t} d\omega$$

$$v_1(t) = \frac{5\pi}{2\pi} \cdot \frac{1}{5} = 0.5$$

$$v_0(t) = v_1(t) + v_2(t) = 0.5 + 0.5\,\text{sgn}(t) - e^{-5t}$$

But $\text{sgn}(t) = -1 + 2u(t)$

$$v_o(t) = +0.5 - 0.5 + u(t) - e^{-5t}u(t) = \underline{\mathbf{u(t) - e^{-5t}u(t)}}$$

Chapter 18, Solution 39.

$$V_s(\omega) = \int_{-\infty}^{\infty} (1-t)e^{-j\omega t}dt = \frac{1}{j\omega} + \frac{1}{\omega^2} - \frac{1}{\omega^2}e^{-j\omega}$$

$$I(\omega) = \frac{V_s(\omega)}{10^3 + j\omega x10^{-3}} = \underline{\frac{10^3}{10^6 + j\omega}\left(\frac{1}{j\omega} + \frac{1}{\omega^2} - \frac{1}{\omega^2}e^{-j\omega}\right)}$$

Chapter 18, Solution 40.

$$\ddot{v}(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$$
$$-\omega^2 V(\omega) = 1 - 2e^{-j\omega} + e^{j\omega 2}$$

$$V(\omega) = \frac{1 - 2e^{-j\omega} + e^{-j\omega 2}}{-\omega^2}$$

Now $Z(\omega) = 2 + \frac{1}{j\omega} = \frac{1+j2\omega}{j\omega}$

$$I = \frac{V(\omega)}{Z(\omega)} = \frac{2e^{j\omega} - e^{j\omega 2} - 1}{\omega^2} \cdot \frac{j\omega}{1 + j2\omega}$$

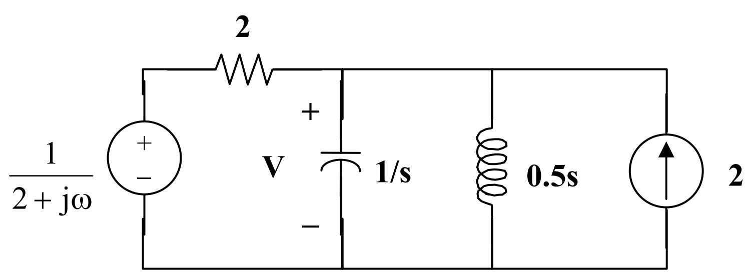
$$= \frac{1}{j\omega(0.5 + j\omega)}\left(0.5 + 0.5e^{-j\omega 2} - e^{-j\omega}\right)$$

But $$\frac{1}{s(s+0.5)} = \frac{A}{s} + \frac{B}{s+0.5} \longrightarrow A = 2, B = -2$$

$$I(\omega) = \frac{2}{j\omega}\left(0.5 + 0.5e^{j\omega 2} - e^{-j\omega}\right) - \frac{2}{0.5 + j\omega}\left(0.5 + 0.5e^{-j\omega 2} - e^{-j\omega}\right)$$

$$i(t) = \underline{\mathbf{\frac{1}{2}sgn(t) + \frac{1}{2}sgn(t-2) - sgn(t-1) - e^{-0.5t}u(t) - e^{-0.5(t-2)}u(t-2) - 2e^{-0.5(t-1)}u(t-1)}}$$

Chapter 18, Solution 41.



$$\frac{V - \frac{1}{2 + j\omega}}{2} + j\omega V + \frac{2V}{j\omega} - 2 = 0$$

$$\left(j\omega - 2\omega^2 + 4\right)V = j4\omega + \frac{j\omega}{2 + j\omega} = \frac{-4\omega^2 + j9\omega}{2 + j\omega}$$

$$V(\omega) = \underline{\frac{2j\omega(4.5 + j2\omega)}{(2 + j\omega)(4 - 2\omega^2 + j\omega)}}$$

Chapter 18, Solution 42.

By current division, $I_o = \frac{2}{2+j\omega} \cdot I(\omega)$

(a) For i(t) = 5 sgn (t),

$$I(\omega) = \frac{10}{j\omega}$$

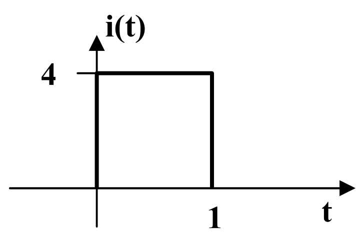
$$I_o = \frac{2}{2+j\omega} \cdot \frac{10}{j\omega} = \frac{20}{j\omega(2+j\omega)}$$

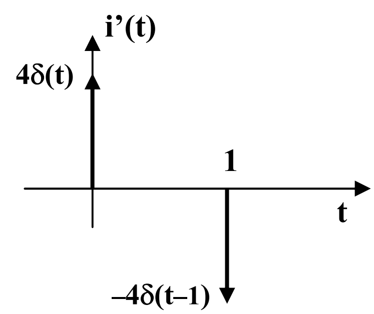
Let $I_o = \frac{20}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$, $A = 10$, $B = -10$

$$I_o(\omega) = \frac{10}{j\omega} - \frac{10}{2+j\omega}$$

$i_o(t)$ = $\underline{\mathbf{5\,sgn(t) - 10e^{-2t}u(t)A}}$

(b)





$$i'(t) = 4\delta(t) - 4\delta(t-1)$$
$$j\omega I(\omega) = 4 - 4e^{-j\omega}$$

$$I(\omega) = \frac{4(1-e^{-j\omega})}{j\omega}$$

$$I_o = \frac{8(1-e^{-j\omega})}{j\omega(2+j\omega)} = 4\left(\frac{1}{j\omega} - \frac{1}{2+j\omega}\right)(1-e^{-j\omega})$$

$$= \frac{4}{j\omega} - \frac{4}{2+j\omega} - \frac{4e^{-j\omega}}{j\omega} + \frac{4e^{-j\omega}}{2+j\omega}$$

$i_o(t) = \underline{\mathbf{2\,sgn(t) - 2\,sgn(t-1) - 4e^{-2t}u(t) + 4e^{-2(t-1)}u(t-1)A}}$

Chapter 18, Solution 43.

$$20\text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j20x10^{-3}\omega} = \frac{50}{j\omega}, \qquad i_s = 5e^{-t} \longrightarrow I_s = \frac{1}{5+j\omega}$$

$$V_o = \frac{40}{40+\frac{50}{j\omega}} I_s \bullet \frac{50}{j\omega} = \frac{50}{(s+1.25)(s+5)}, \qquad s = j\omega$$

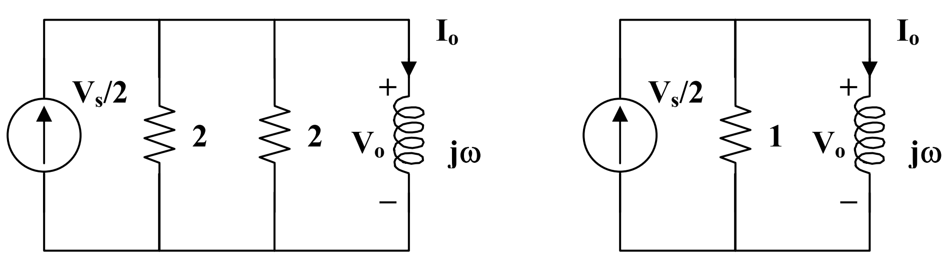
$$V_o = \frac{A}{s+1.25} + \frac{B}{s+5} = \frac{40}{3}\left[\frac{1}{j\omega+1.25} - \frac{1}{j\omega+5}\right]$$

$$v_o(t) = \underline{\frac{40}{3}(e^{-1.25t} - e^{-5t})u(t)}$$

Chapter 18, Solution 44.

$$1\text{H} \longrightarrow j\omega$$

We transform the voltage source to a current source as shown in Fig. (a) and then combine the two parallel 2Ω resistors, as shown in Fig. (b).



(a) (b)

$$2\|2 = 1\Omega, \quad I_o = \frac{1}{1+j\omega}\cdot\frac{V_s}{2}$$

$$V_o = j\omega I_o = \frac{j\omega V_s}{2(1+j\omega)}$$

$$\ddot{v}_s(t) = 10\delta(t) - 10\delta(t-2)$$
$$j\omega V_s(\omega) = 10 - 10e^{-j2\omega}$$

$$V_s(\omega) = \frac{10\left(1 - e^{-j2\omega}\right)}{j\omega}$$

Hence $V_o = \dfrac{5\left(1 - e^{-j2\omega}\right)}{1 + j\omega} = \dfrac{5}{1 + j\omega} - \dfrac{5}{1 + j\omega}e^{-j2\omega}$

$$v_o(t) = 5e^{-t}u(t) - 5e^{-(t-2)}u(t-2)$$
$$v_o(1) = 5e^{-1} - 1 - 0 = \underline{\mathbf{1.839\ V}}$$

Chapter 18, Solution 45.

$$V_o = \frac{\dfrac{1}{j\omega}}{2 + j\omega + \dfrac{1}{j\omega}}(2) = \frac{2}{(j\omega + 1)^2} \longrightarrow \underline{v_o(t) = 2te^{-t}u(t)}$$

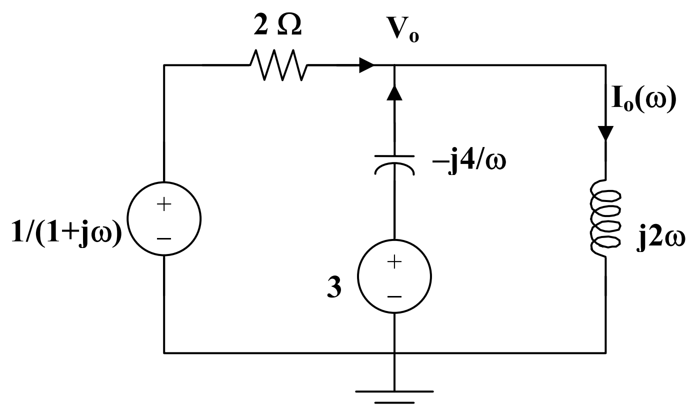
Chapter 18, Solution 46.

$$\frac{1}{4}F \longrightarrow \frac{1}{j\omega\frac{1}{4}} = \frac{-j4}{\omega}$$

$$2H \longrightarrow j\omega 2$$
$$3\delta(t) \longrightarrow 3$$

$$e^{-t}u(t) \longrightarrow \frac{1}{1 + j\omega}$$

The circuit in the frequency domain is shown below:



At node V_o, KCL gives

$$\frac{\frac{1}{1+j\omega} - V_o}{2} + \frac{3 - V_o}{\frac{-j4}{\omega}} = \frac{V_o}{j2\omega}$$

$$\frac{2}{1+j\omega} - 2V_o + j\omega 3 - j\omega V_o = -\frac{j2V_o}{\omega}$$

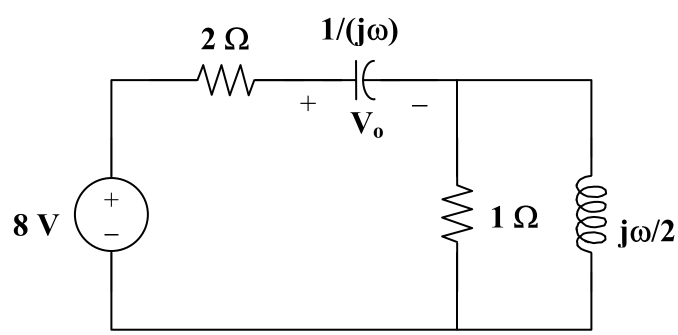
$$V_o = \frac{\frac{2}{1+j\omega} + j\omega 3}{2 + j\omega - \frac{j2}{\omega}}$$

$$I_o(\omega) = \frac{V_o}{j2\omega} = \frac{\frac{2 + j\omega 3 - 3\omega^2}{1+j\omega}}{j2\omega\left(2 + j\omega - \frac{j2}{\omega}\right)}$$

$$\underline{\mathbf{I_o(\omega) = \frac{2 + j\omega^2 - 3\omega^2}{4 - 6\omega^2 + j(8\omega - 2\omega^3)}}}$$

Chapter 18, Solution 47.

Transferring the current source to a voltage source gives the circuit below:



Let $Z_{in} = 2 + 1 \| \frac{j\omega}{2} = 2 + \frac{\frac{j\omega}{2}}{1 + \frac{j\omega}{2}} = \frac{4 + j3\omega}{2 + j\omega}$

By voltage division,

$$V_o(\omega) = \frac{\frac{1}{j\omega}}{\frac{1}{j\omega} + Z_{in}} \cdot 8 = \frac{8}{1 + j\omega Z_{in}} = \frac{8}{1 + \frac{j\omega(4 + j3\omega)}{2 + j\omega}}$$

$$= \frac{8(2 + j\omega)}{2 + j\omega + j\omega 4 - 3\omega^2}$$

$$= \underline{\mathbf{\frac{8(2 + j\omega)}{2 + j\omega 5 - 3\omega^2}}}$$

Chapter 18, Solution 48.

$$0.2F \longrightarrow \frac{1}{j\omega C} = -\frac{j5}{\omega}$$

As an integrator,

$$RC = 20 \, x \, 10^3 \, x \, 20 \, x \, 10^{-6} = 0.4$$

$$v_o = -\frac{1}{RC}\int_o^t v_i dt$$

$$V_o = -\frac{1}{RC}\left[\frac{V_i}{j\omega} + \pi V_i(0)\delta(\omega)\right]$$

$$= -\frac{1}{0.4}\left[\frac{2}{j\omega(2 + j\omega)} + \pi\delta(\omega)\right]$$

$$I_o = \frac{V_o}{20} mA = -0.125\left[\frac{2}{j\omega(2 + j\omega)} + \pi\delta(\omega)\right]$$

$$= -\frac{0.125}{j\omega} + \frac{0.125}{2 + j\omega} - 0.125\pi\delta(\omega)$$

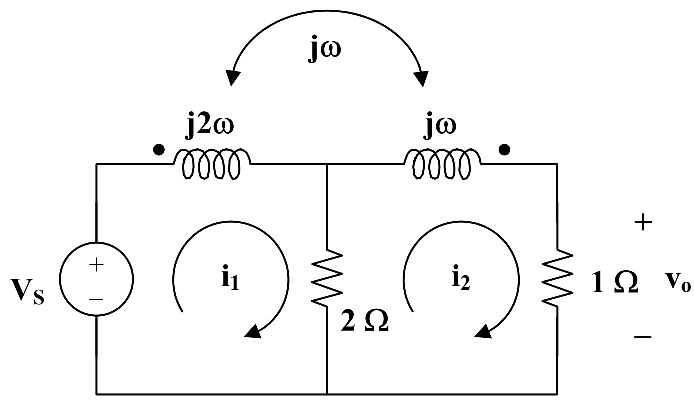
$$i_o(t) = -0.125\,\text{sgn}(t) + 0.125e^{-2t}u(t) - \frac{0.125}{2\pi}\int \pi\delta(\omega)e^{j\omega t}dt$$

$$= 0.125 + 0.25u(t) + 0.125e^{-2t}u(t) - \frac{0.125}{2}$$

$$i_o(t) = \underline{\mathbf{0.625 - 0.25u(t) + 0.125e^{-2t}u(t)\,mA}}$$

Chapter 18, Solution 49.

Consider the circuit shown below:



$$V_s = \pi[\delta(\omega+1) + \delta(\omega-2)]$$

For mesh 1, $-V_s + (2 + j2\omega)I_1 - 2I_2 - j\omega I_2 = 0$

$$V_s = 2(1 + j\omega)I_1 - (2 + j\omega)I_2 \qquad (1)$$

For mesh 2, $0 = (3 + j\omega)I_2 - 2I_1 - j\omega I_1$

$$I_1 = \frac{(3+\omega)I_2}{(2+\omega)} \qquad (2)$$

Substituting (2) into (1) gives

$$V_s = 2\frac{2(1+j\omega)(3+j\omega)I_2}{2+j\omega} - (2+j\omega)I_2$$

$$V_s(2+\omega) = [2(3 + j4\omega - \omega^2) - (4 + j4\omega - \omega^2)]I_2$$
$$= I_2(2 + j4\omega - \omega^2)$$

$$I_2 = \frac{(s+2)V_s}{s^2+4s+2},\ s = j\omega$$

$$V_o = I_2 = \frac{(j\omega+2)\pi[\delta(\omega+1)+\delta(\omega-1)]}{(j\omega)^2+j\omega4+2}$$

$$v_o(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} v_o(\omega)e^{j\omega t}d\omega$$

$$= \int_{-\infty}^{\infty}\frac{\frac{1}{2}(j\omega+2)e^{j\omega t}\delta(\omega+1)d\omega}{(j\omega)^2+j\omega4+2} + \frac{\frac{1}{2}(j\omega+2)e^{j\omega t}\delta(\omega-1)d\omega}{(j\omega)^2+j\omega4+2}$$

$$= \frac{\frac{1}{2}(-j+2)e^{jt}}{-1-j4+2} + \frac{\frac{1}{2}(j+2)e^{jt}}{-1+j4+2}$$

$$v_o(t) = \frac{\frac{1}{2}(2-j)(1+j4)}{17}e^{jt} + \frac{\frac{1}{2}(2-j)(1-j4)e^{jt}}{17}$$

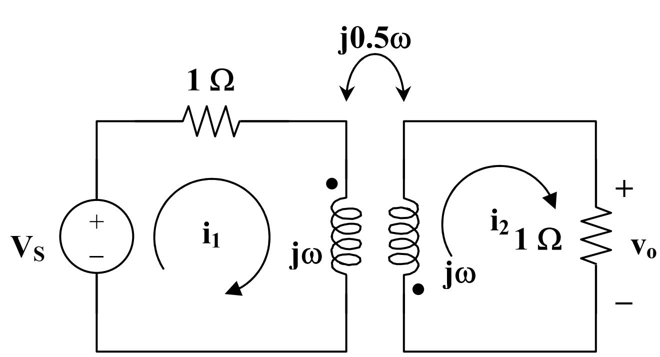
$$= \frac{1}{34}(6+j7)e^{jt} + \frac{1}{34}(6-j7)e^{jt}$$

$$= 0.271e^{-j(t-13.64°)} + 0.271e^{j(t-13.64°)}$$

$v_o(t) =$ $\underline{\mathbf{0.542\cos(t-13.64°)V}}$

Chapter 18, Solution 50.

Consider the circuit shown below:



For loop 1,

$$-2+(1+j\omega)I_1 + j0.5\omega I_2 = 0 \quad (1)$$

For loop 2,

$$(1+j\omega)I_2 + j0.5\omega I_1 = 0 \quad (2)$$

From (2),

$$I_1 = \frac{(1+j\omega)I_2}{-j0.5\omega} = -2\frac{(1+j\omega)I_2}{j\omega}$$

Substituting this into (1),

$$2 = \frac{-2(1+j\omega)I_2}{j\omega} + \frac{j\omega}{2}I_2$$

$$2j\omega = -\left(4 + j4\omega - \frac{3}{2}\omega^2\right)I_2$$

$$I_2 = \frac{2j\omega}{4 + j4\omega - 1.5\omega^2}$$

$$V_o = I_2 = \frac{-2j\omega}{4 + j4\omega + 1.5(j\omega)^2}$$

$$V_o = \frac{\frac{4}{3}j\omega}{\frac{8}{3} + j\frac{8\omega}{3} + (j\omega)^2}$$

$$= \frac{-4\left(\frac{4}{3} + j\omega\right)}{\left(\frac{4}{3} + j\omega\right)^2 + \left(\frac{\sqrt{8}}{3}\right)^2} + \frac{\frac{16}{3}}{\left(\frac{4}{3} + j\omega\right)^2 + \left(\frac{\sqrt{8}}{3}\right)^2}$$

$$V_o(t) = \underline{\mathbf{-4e^{-4t/3}\cos\left(\frac{\sqrt{8}}{3}t\right)u(t) + 5.657e^{-4t/3}\sin\left(\frac{\sqrt{8}}{3}t\right)u(t)\,V}}$$